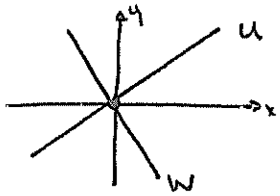
Solution

Math 108A - Quiz # 2

April 29, 2009

1. True or False: If U and W are subspaces of a vector space V and $\dim U = \dim W$, then $U = W$.

FALSE! Consider 2 lines in $\mathbb{R}^2$. They both have dimension 1, but are not equal.



counterexample: $U = \mathbb{R}(0,1)$ = y-axis

$W = \mathbb{R}(1,0)$ = x-axis

$\dim U = \dim W = 1$

but $U \neq W$.

2. Find a basis for the subspace

$$U = \{(a, b, a, c) \mid a, b, c \in F\} \subseteq F^4$$

$U = \{a(1,0,1,0) + b(0,1,0,0) + c(0,0,0,1) \mid a,b,c \in F\}$

$= \text{Span}\{(1,0,1,0), (0,1,0,0), (0,0,0,1)\}$.

$\{(1,0,1,0), (0,1,0,0), (0,0,0,1)\}$ is also a basis for U

Since it is linearly independent.

$((a,b,a,c) = \vec{0} \Rightarrow a = b = c = 0)$

3. State the definition of a **linear map** T from V to W. (Equivalent forms of the definition are also acceptable.)

A Linear Map T from V to W is

a function $T : V \longrightarrow W$ such that

$\forall a \in F$ and $\forall \vec{u}, \vec{v} \in V$

1) $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$

and 2) $T(a\vec{u}) = a\,T(\vec{u})$.