Math 108A - Home Work # 5

Due: May 16, 2008

1. LADR p. 60-61: Exercises 16, 24.

2. Give an example of two vector spaces V and W and two linear maps $T : V \to W$ and $S : W \to V$ such that $ST = I_V$ but $TS \neq I_W$. In your example, is either of S, T injective? Is either surjective?

3. Let $T : V \to W$ be a linear map, and let $\{v_1, \ldots, v_n\}$ be a basis for V. Show that T is invertible if and only if $\{Tv_1, \ldots, Tv_n\}$ is a basis for W. (You can use questions from the previous homework (eg., 5 and 7 on p. 59) to shorten your argument.)

4. Let A be an $n \times n$ matrix with entries in F.

   (a) Show that A is invertible if and only if its columns are linearly independent (column) vectors in F^n. (Since A has n columns and $n = \dim F^n$, we could also say that A is invertible if and only if its columns are a basis of F^n.) Hint: this is a consequence of the previous exercise.

   (b) Show that A is invertible if and only if its rows are linearly independent vectors in F^n. (Here, it might be easier to replace "A is invertible" with "A is surjective" and note why these are equivalent.)