

Math 116 - Final Exam Review Sheet

Below is an outline of the topics we've covered this quarter, along with suggestions of problems to practice or review from the text (some are old homework problems). The Final Exam will be cumulative, so you should study everything. However, the questions on the test will not require you to use a specific method. For instance, to solve a recurrence relation you can use Theorem 7.2.1 or use generating functions; and to prove identities for binomial coefficients you can use a combinatorial argument or the binomial theorem. The test will cover material from chapters 3, 5, 6, 7 and 8 of the text, as indicated in the assigned readings on the course webpage.

- **Basic Counting Principles (3.1).**

- Know how and when to use the Addition, Multiplication, Subtraction and Division Principles.
- 3.6: Ex. 4, 1, 6

- **Counting Combinations and Permutations (3.2 - 3.3).**

- Know the formulas for $P(n, k)$ and $C(n, k)$, and when to use them. The most important thing to keep in mind here is that Permutations are ORDERED arrangements, while Combinations are UNORDERED arrangements.
- Know how to count circular permutations (it often helps to fix the position of one of the objects).
- 3.6: Ex. 7, 8, 12, 13, 14, 15,

- **Counting Permutations and Combinations with Repetition (3.4-3.5).**

- Theorems 3.4.1, 3.4.2, and 3.5.1 are the most important. One example of Permutations with Repetition consists of all the rearrangements of the letters in MISSISSIPPI. An example of Combinations with Repetition consists of all the ways to choose 12 donuts of 4 different types. You should also be comfortable interpreting Combination-with-Repetition problems in terms of nonnegative integer solutions to equations of the form $x_1 + \cdots + x_k = r$.
- Theorem 3.4.3 describes how to count the number of ways of dividing people up into unlabeled teams.
- 3.6: Ex. 55, 31, 37, 38

- **Counting Combinations with Repetition and Restrictions (6.2, 7.4).**

- One method involves the Inclusion-Exclusion Principle (when the restrictions are upper bounds on the repetition numbers).

- The other method is to use ordinary generating functions. This works better if the restrictions are more complicated (i.e., if one object must be chosen an odd number of times), or if you want to find a formula for the number of n -combinations in terms of n .
- 6.7: Ex. 6, 8; 7.8: Ex. 29, 34, 35.

- **Pascal's Triangle, Binomial Coefficients (5.1, 5.3).**

- Know Pascal's triangle and how it relates to the binomial coefficients. Pascal's Identity.
- Be able to prove identities for the binomial coefficients using EITHER (a) induction, OR (b) combinatorial arguments. (You will have a choice on the exam.)
- 5.8: Ex. 10, 25, Try proving (5.6) and (5.7) by induction on n .

- **Binomial Theorem (5.2-5.3).**

- Know the Binomial Theorem, and how to use it to prove identities for the binomial coefficients. The latter can often be done by replacing x and y with numbers, or else by first integrating/differentiating with respect to x and then plugging in numbers for x and y .
- 5.8: Ex. 7, 8, 9, 15, 18.

- **Inclusion-Exclusion Principle (6.1-6.2).**

- Know the statement of the IEP. It has 2 forms: Theorem 6.1.1 and Corollary 6.1.2, which are equivalent by the Subtraction Principle.
- Applications to counting combinations with repetition (see above).
- 6.7: Ex. 2, 3.

- **Derangements (6.3).**

- Know the definition of and formula for D_n (Theorem 6.3.1).
- Know the recurrence relations for D_n (6.5) and (6.7).
- Be able to count permutations that are only partial derangements, or that are somehow related to derangements.
- 6.7: Ex. 11, 12, 13, 16.

- **Recursively Defined Sequences (7.1-7.2)**

- Be able to use induction to prove properties of a recursively defined sequence (such as the Fibonacci numbers).
- Be able to show that a sequence that is defined combinatorially satisfies a certain recurrence relation.

- Be able to prove that two sequences are equal by showing that they satisfy the same recurrence relation and have the same initial values.
- Be able to solve a linear homogeneous recurrence relation with initial values (Theorems 7.2.1 and 7.2.2)
- 7.8: Ex. 1a, 3a, 11, 14, 19c, 20.

- **Generating Functions (7.4, 7.5, 7.7).**

- Know the definitions of generating functions and be able to find an explicit formula for the generating function of a given sequence. It may help to know a few examples as in (7.4).
- Applications to counting combinations with repetition and restrictions (see above).
- Applications to solving homogeneous linear recurrence relations (7.5).
- Exponential generating functions: Know the definition and applications to counting permutations with repetition and restrictions.
- 7.8: Ex. 28, 30d, 36, 37, 42, 45.

- **Catalan Numbers (8.1, 7.6).**

- Know the formula and recurrence relation for the Catalan numbers, as well as one of the equivalent definitions in terms of subdiagonal up-right paths or sequences of ± 1 's with nonnegative partial sums.
- You should be able to relate another sequence to the Catalan numbers by showing that it satisfies the same recurrence relation, or by showing that what it counts is in one-to-one correspondence with the subdiagonal up-right paths from $(0,0)$ to (n,n) .
- 8.6: Ex. 1, 2.

- **Difference Sequences (8.2).**

- Use Theorem 8.2.2 to find the formula of a polynomial with certain given values.
- 8.6: Ex. 6, 7, 8 (Theorem 8.2.3 may be useful.)