Math 5C, Homework #5

Due: February 14, 2007

1. For each of the following sequences, find a formula for the n^{th} term a_n in terms of n, decide whether the sequence converges or diverges, and compute $\lim_{n\to\infty} a_n$ if it converges.

   (a) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$

   (b) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$

   (c) $\frac{5}{2}, \frac{1}{2}, \frac{1}{10}, \frac{1}{50}, \ldots$

   (d) $0, \frac{3}{2}, \frac{3}{4}, \frac{9}{8}, \frac{15}{16}, \frac{33}{32}, \ldots$

2. For each of the following series, compute the first 5 partial sums (or more), and use them to make a guess about the value of the infinite sum (if it appears to converge).

   (a) $\sum_{n=1}^{\infty} 3^{-n}$

   (b) $\sum_{n=0}^{\infty} \frac{4(-1)^n}{2n+1}$

   (c) $\sum_{n=0}^{\infty} \frac{1}{n!}$ (Recall that $n!$, read "n factorial", is the product $n(n-1)(n-2)\cdots 2\cdot 1$ of all the integers between 1 and n, and $0! = 1$)

3. Let $a_n = \ln(1 + \frac{1}{n})$ for $n \geq 1$.

   (a) Does the sequence $\{a_n\}_{n\geq 1}$ converge? If so, what is its limit?

   (b) Does $\sum_{n=1}^{\infty} a_n$ converge or diverge? (Hint: can you write a_n as a difference of two things?)

4. A sequence $\{a_n\}_{n\geq 1}$ can be defined by the following rules: $a_1 = 1$ and $a_{n+1} = \frac{1}{a_n+1}$ for all $n \geq 1$ (For example, if $a_{16} = 20$ then $a_{17} = 1/(20+1) = 1/21$).

   (a) Calculate the first 5 terms of this sequence. Does it appear to converge to a limit?

   (b) Assuming that the sequence does converge, find its limit. (Hint: you may use the fact that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.)

   (c) (Extra Credit): How is this sequence related to the Fibonacci numbers and (more importantly) why?