Math 5C, Midterm 1 Review Problems

Winter 2007

1. Compute $\int_C yz\ dx + 2x\ dy - y\ dz$ where C is the straight line path from $(1,2,1)$ to $(-1,3,0)$.

2. Find the surface area of the surface S, which is parametrized by

$$\phi(u,v) = \begin{cases} x(u,v) = u - v \\ y(u,v) = u + v \\ z(u,v) = uv \end{cases}$$

for all (u,v) with $u^2 + v^2 \leq 1$.

3. Let S be the top half of the unit sphere (i.e., S is given by $x^2+y^2+z^2 = 1$ and $z \geq 0$), oriented by the outer normal. Integrate

$$\int\int_S x\ dy\ dz + y\ dz\ dx + z^2\ dx\ dy.$$

4. Let S be the surface given by $z = xy^2 - 3x^2$ with upper normal $\mathbf{n}$, over the square with vertices $(\pm 1, \pm 1)$ in the xy-plane. If $\mathbf{w} = (z + 3x^2)\mathbf{i} + yz\mathbf{j} + y^2\mathbf{k}$, calculate

$$\int\int_S \mathbf{w} \cdot \mathbf{n}\ d\sigma.$$

5. Let S be the unit sphere, $x^2+y^2+z^2 = 1$, oriented outward, and let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y,z) = xy^2\ \mathbf{i} - xz^2\ \mathbf{j} + x^2z\ \mathbf{k}$. Use the Divergence Theorem to compute $\int\int_S \mathbf{F} \cdot \mathbf{n}\ d\sigma$.

6. Let S be the cone $x^2 = y^2 + z^2$, $0 \leq x \leq 2$, oriented inward (so the normal vectors point toward the x-axis). Use Stokes' Theorem to calculate $\int\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\ d\sigma$, where $\mathbf{F}(x,y,z) = x^2\mathbf{i} - z\mathbf{j} + (y^2 - z)\mathbf{k}$.

7. Let C be the curve given by $x = \sin t$, $y = \cos t$, $z = \cos(2t)$ for $0 \leq t \leq 2\pi$. Use Stokes' Theorem to evaluate

$$\oint_C xz\ dx + y^2\ dy + z^2\ dz.$$

8. Show that the integral

$$\int_{(-1,1,3)}^{(\pi/2,0,1)} z^2 \cos(x+y^2)\ dx + 2yz^2\cos(x+y^2)\ dy + 2z\sin(x+y^2)\ dz$$

is independent of path and evaluate it.

9. Let $\mathbf{u}$ be the vector field

$$\mathbf{u}(x, y, z) = \frac{y}{x^2 + y^2}\mathbf{i} - \frac{x}{x^2 + y^2}\mathbf{j} + z^2\mathbf{k}$$

on $\mathbb{R}^3$ minus the z-axis.

(a) Show that $\operatorname{curl}(\mathbf{u}) = \mathbf{0}$ on this domain.

(b) Show that $\mathbf{u}$ is not the gradient vector field of any function F on this domain. (Hint: Find a closed curve C with $\int_C u_T \, ds \neq 0$.)