Math 5C, Midterm 2 Review Problems

Winter 2007

1. The first several terms of a sequence are given. Find a formula in terms of n for the n^{th} term of the sequence (be sure to say what value n starts at). Then find the limit of the sequence as n tends towards infinity, if one exists.

   $(a)\ -4, 1, \frac{-1}{4}, \frac{1}{16}, \dots$ $(b)\ \frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \dots$

   $(c)\ 1, -3, 8, -24, 35, -48, 63 \dots$ $(d)\ 1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \frac{9}{16}, \dots$

2. Find the sum of the infinite series, or show that it diverges:

   $(a)\ \sum_{n=1}^{\infty} 2^{1-2n}$ $\sum_{n=1}^{\infty} \left(\frac{\sqrt{2n+1}}{n} - \frac{\sqrt{2n+3}}{n+1} \right)$

3. Do the following series converge or diverge? Justify your answers.

   $(a)\ \sum_{n=1}^{\infty} \sqrt{\frac{n+1}{3n^2}}$ $(b)\ \sum_{n=2}^{\infty} \frac{1}{n \ln n}$

   $(c)\ \sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ $(d)\ \sum_{n=1}^{\infty} \frac{n^n}{2 \cdot 4 \cdot 6 \cdots (2n)}$

4. Find the interval of convergence of the following power series:

   $(a)\ \sum_{n=0}^{\infty} \frac{(x-3)^n}{4^n}$ $\sum_{n=0}^{\infty} \frac{x^{2n}}{n+1}$

5. Suppose that the power series $\sum_{n=0}^{\infty} c_n (x-1)^n$ converges when $x = 2$ and diverges when $x = -3$. For each of the following values of x, state whether the series converges or diverges, or whether you can't tell: (a) $x = 1$; (b) $x = 0$; (c) $x = 1/2$; (d) $x = 3$; (e) $x = 5$; (f) $x = 6$.

6. Find the MacLaurin series and its interval of convergence for the function $f(x) = \frac{x^2}{(1+x)^2}$.

7. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$. (Hint: This sum can be realized as the MacLaurin series of some function $f(x)$ evaluated at a certain value of x.)