Final Exam
Math 5C, UCSB, Fall '06

You have 3 hours to complete this exam.

Name:____________________

Perm #:____________________

Signature:____________________

Discussion section:____________________

Show all your work. Partial credit will be given only if work is relevant and correct. *Please make your work as clear and easy to follow as possible.* You might want to put scratch work on the back of every sheet, and put neat clean work on the front of every sheet. You don't need to simplify your answers but you need to justify them.

Problem	Possible Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Extra	10	
Total	50 (+10)	

Name:______________________________

Exercise 1.

Verify Stoke's theorem for the vector field $\mathbf{F} = (xy, xz, z+1)$ where S is the surface defined by $z = x^2 + y^2$ and $z = 1$.

Name:________________________________

Exercise 2.

Find the Fourier series of the function

$$f(x) = \begin{cases} 2 & \text{if } 2k\pi \le x < (2k+1)\pi \\ 0 & \text{if } (2k+1)\pi \le x < (2k+2)\pi \end{cases}$$

where $k = 0, \pm 1, \pm 2, \ldots$.

$f(x) =$ ________________________________

Name:________________________________

Exercise 3.

Let S be the upper half surface of the sphere $x^2 + y^2 + z^2 = 4$. Compute

$\int\int_S z^2 \, dS =$ ________________________________

Name:____________________________________

Exercise 4.

Expand the function $f(x) = x^2 e^{2x}$ into a power series around the point $x = 0$.

Name:________________________________

Exercise 5.

Determine wether the following series converge (Justify your answer):

1) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{2k+1}$

2) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{e^{2k}-1}$

3) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2k^2-1}$

Name:________________________________

Exercise 6.

Find the solution for the wave equation:

$$\begin{cases} u_{tt} - 25\ u_{xx} = 0 & \\ u(0,t) = 0 & t \geq 0 \\ u(\pi,t) = 0 & t \geq 0 \\ u(x,0) = 2\ \sin(x) & 0 \leq x \leq \pi \\ u_t(x,0) = 3\ \sin(2x) & 0 \leq x \leq \pi \end{cases}$$

$u(x,t) =$ ________________________________

Name:______________________________

Exercise 7.

Determine for which x the following series converges:

$$\sum_{k=1}^{\infty} e^{kx}$$

Compute its value for all the values of x, for which it converges.

Name: ______________________________________

Exercise 8. (Extra)

Let S be the surface defined by $z = x^2 + y^2$, $z = -x^2 - y^2$ and $x^2 + y^2 = 1$ and let $\mathbf{F} = (x + y, y + z, x^2 + z)$.

Compute

$$\int\int_S \mathbf{F}_n \, d\sigma = \underline{\hspace{5cm}}$$

Name:____________________________