Definition A set S is countably infinite if $S \approx \mathbb{N}$.
An infinite set S is uncountable if $S \not\approx \mathbb{N}$.

eg $A = \{a, 13, \emptyset, \{\emptyset\}\}$ is finite. $|A| = 4$.
$\mathbb{Z}$ is countably infinite (by the previous example)
$\mathbb{R}$ is uncountable (to be proved later)

Thm (Pigeonhole Principle) Let A & B be nonempty finite sets. If $|A| > |B|$ then there does not exist an injection $f: A \to B$.

(ie. If you have n pigeonholes and $n+1$ pigeons in the pigeonholes, then some pigeonhole contains at least 2 pigeons)

(See text 4.8 & 4.9 pp. 155-156 for a proof)

Corollary $\mathbb{N}$ is infinite.
proof By contradiction. Assume $\mathbb{N} \approx \mathbb{N}_n$ for some $n \geq 0$.
Let $f: \mathbb{N} \to \mathbb{N}_n$ be a bijection.
Let $m \in \mathbb{N}$ w/ $m > n$.
Then $f(1), f(2), \ldots, f(m)$ are m distinct elements in $\mathbb{N}_n$ = contradiction.
(We have produced an injection $\mathbb{N}_m \to \mathbb{N}_n$ by restricting f to the subset $\{1, 2, \ldots, m\} \subseteq \mathbb{N}$. Such an injection cannot exist by the above theorem.)

Thm a) If $A \subseteq B$ and B is finite, then A is finite and $|A| \leq |B|$.
b) If $A \subseteq B$ and A is infinite, then B is infinite.
c) A is infinite $\iff \exists f: \mathbb{N} \to A$ injective.
(Obvious: But see Text 4.11, 4.12 pp. 158-159 for proofs)