Name: ________________________

Practice Problems

Professor: Paul J. Atzberger

Introduction to Numerical Analysis, 104A

Scoring:

Problem1: ____________

Problem2: ____________

Problem3: ____________

Problem4: ____________

Directions: Answer each question carefully and be sure to show all of your work. You are permitted to use a calculator but please be sure to show intermediate steps in your calculations. If you have any questions please feel free to ask.

Problem 1: Compute to four decimal places the absolute and relative errors when approximating p by the value p^*. Also, state the number of significant digits.

a) $p = e$ by $p^* = 2.718$

b) $p = e^{-1}$ by $p^* = 1839/5000$

Problem 2: Consider the following two formulas for computing a value p.
Formula 1: $p_1^* = ((e^1 \cdot \pi + \sqrt{6} \cdot \pi) + e^1\sqrt{6}) + 6$.

Formula 2: $p_2^* = (\sqrt{6} + \pi) \cdot (\sqrt{6} + e^1)$.

The exact value is approximately $p = 28.89344$.

(a) To model round-off errors use 2-digit-chopping for each formula to compute the numerical value p^*. Please be sure to show your intermediate steps and your work carefully.

(b) What is is the absolute error and the relative error of the value p^* obtained from each formula?

(c) What is the number of significant digits in your final solution using each formula?

(d) For numerically computing p which of the formulas is more robust to round-off errors? Why?

Problem 3: Compute an approximation after three iterations of the bisection method to the solution of $f(x) = x^3 - x^2 + 2 = 0$ when starting with $a = -3$ and $b = 3$.

(a) State the relative and absolute errors for the root $x = -1$.

(b) At most how many iterations would be required to approximate using the bisection method the root with an accuracy of 10^{-2}?

Problem 4: For the following fixed-point iteration methods $x_{n+1} = g(x_n)$ determine the fixed point and the rate of convergence. Use that for x_n close to the fixed-point p we have $|x_n - p| \approx k|x_{n-1} - p| \approx k^n|x_0 - p|$ with $|g'(x)| < k$.

a) $g(x) = x - \frac{1}{100}(x^3 - 2x)$, with $|x_0| < 1$.

b) $g(x) = x - \frac{(x^2-2)}{2x}$, with $|x_0| < 2$.