Name: _____

Midterm Exam: Professor: Paul J. Atzberger Introduction to Numerical Analysis, 104A Sample Exam

Scoring:	
Problem1:	
Problem2:	
Problem3:	

Problem4: _____

Directions: Answer each question carefully and be sure to show all of your work. You are permitted to use a calculator but please be sure to show intermediate steps in your calculations. If you have any questions please feel free to ask.

Problem 1: Compute to four decimal places the absolute and relative errors when approximating p by the value p^* . Also, state the number of significant digits.

a) p = e by $p^* = 2.718$

b)
$$p = e^{-1}$$
 by $p^* = 1839/5000$

Problem 2: Consider the following two formulas for computing a value p. Formula 1: $p_1^* = ((e^1 \cdot \pi + \sqrt{6} \cdot \pi) + e^1\sqrt{6}) + 6$.

Formula 2: $p_2^* = (\sqrt{6} + \pi) \cdot (\sqrt{6} + e^1).$

The exact value is approximately p = 28.89344.

(a) To model round-off errors use 2-digit-chopping for each formula to compute the numerical value p^* . Please be sure to show your intermediate steps and your work carefully.

(b) What is is the absolute error and the relative error of the value p^* obtained from each formula?

(c) What is the number of significant digits in your final solution using each formula?

(d) For numerically computing p which of the formulas is more robust to round-off errors? Why?

Problem 3: Compute an approximation after three iterations of the bisection method to the solution of $f(x) = x^3 - x^2 + 2 = 0$ when starting with a = -3 and b = 3.

(a) State the relative and absolute errors for the root x = -1.

(b) At most how many iterations would be required to approximate using the bisection method the root with an accuracy of 10^{-2} ?

Problem 4: For the following fixed-point iteration methods $x_{n+1} = g(x_n)$ determine the fixed point and the rate of convergence. Use that for x_n close to the fixed-point p we have $|x_n - p| \approx k |x_{n-1} - p| \approx k^n |x_0 - p|$ with |g'(x)| < k.

a)
$$g(x) = x - \frac{1}{100}(x^2 - 2)$$
, with $x_0 = 0.5$.

b)
$$g(x) = x - \frac{(x^2 - 2)}{2x}$$
, with $x_0 = 0.5$.