



# Final Exam Outline

Math 124B: Numerical Analysis

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- Fourier Series for functions on an Interval  $[-\ell, \ell]$ 
  - real-valued sine/cosine expansion
    - coefficients from function integrations
    - series representations
  - complex-valued exponential expansion
    - coefficients from function integrations
    - series representations
  - conversion between
    - real-valued  $A_n, B_n$  coefficients
    - complex-valued coefficients  $c_n$
- Analysis of Fourier Series
  - definitions of convergence
    - uniform convergence
    - pointwise convergence
    - $L^2$ -convergence
    - weak convergence
  - theorems for convergence (conditions)
    - uniform convergence
    - pointwise convergence
    - $L^2$ -convergence
  - ability to compute fourier series representations of
    - continuous functions
    - discontinuous functions
    - $L^2$ -functions
- Solution of Parabolic PDEs
  - Fourier series approaches in the following cases
    - periodic boundary conditions.
    - homogeneous dirichlet boundary conditions.
    - homogeneous neumann boundary conditions.
    - inhomogeneous dirichlet boundary conditions,  $h(t)$ ,  $j(t)$ .
- Solution of Hyperbolic PDEs
  - Fourier series approaches in the following cases
    - periodic boundary conditions.
    - homogeneous dirichlet boundary conditions.
- Solution of Elliptic PDEs on Rectangles and Cubes
  - maximum principle
  - existence and uniqueness

- Fourier series approaches in the following cases
  - rectangle: dirichlet boundary conditions (homogeneous).
  - rectangle: neumann boundary conditions (homogeneous).
  - cube: dirichlet boundary conditions (homogeneous).
  - cube: neumann boundary conditions (homogeneous).
- Solution of Elliptic PDEs on Disks and Wedges
  - poisson formula for the disk
    - dirichlet boundary conditions
    - mean-value property
    - maximum principle
  - fourier series approaches for the wedge
    - dirichlet boundary conditions
    - neumann boundary conditions
- Discrete Fourier Transform (DFT)
  - definition of the (DFT) and inverse (IDFT)
  - aliasing formula
  - fourier interpolation using DFTs
  - solution of elliptic PDEs on an interval with periodic conditions
- Approximate Solutions using Finite Difference Methods for PDEs
  - von Neumann Analysis
  - stability of finite difference methods
  - approximating solutions of parabolic and hyperbolic PDEs