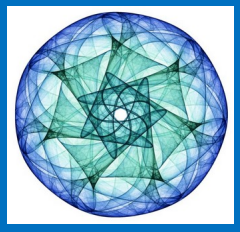

Final Exam Outline

Math 124A: Numerical Analysis
Professor: Paul J. Atzberger

(look for any updates near end of the quarter)

- First-Order PDEs and Method of Characteristics
  - constant coefficient case
    - form of the characteristics (lines)
    - general solutions
  - variable coefficient case when $a(x,y)>0$
    - characteristic equations
    - form of the characteristics based on $y(x)$
    - general solutions
  - general variable coefficient case
    - characteristic equations for $y(s)$
    - general solutions
  - inhomogeneous case
    - method of characteristics with sources/sinks
  - examples
    - transport PDEs

- Second-Order PDEs
  - linear change-of-variables
    - relationship between the gradients in each coordinate system
    - how second-order differential operators transform under coordinate changes
  - classifying second-order PDEs
    - expressing PDE similar to a quadratic form in matrix-vector notation
    - elliptic case
      - conditions on the coefficients a_{ij}
      - canonical form when first-order terms are zero
    - hyperbolic case
      - conditions on the coefficients a_{ij}
      - canonical form when first-order terms are zero
    - parabolic case
      - conditions on the coefficients a_{ij}
      - canonical form when all but one first-order term is zero
  - definition of well-posedness.
  - sufficient criteria for well-posedness.
  - uniqueness, existence, robustness to perturbations.

- Hyperbolic PDEs
  - wave equation initial value problem
  - wave equation with a source
  - solution technique in 1D on R^1

- form of the general solution (D'Almbert's Formula)
- example
  - hat function initial conditions
- domain of dependence
- domain of influence
- kinetic and potential energy of solutions
- conservation of energy principle

- Parabolic PDEs
  - diffusion equation initial value problem
  - diffusion equation with a source
  - uniqueness
  - stability
  - solution technique in 1D on R^1
  - properties of the diffusion equation
  - special solution (Green's function)
  - form of the general solution
  - entropy production
  - maximum principle

- Boundary Conditions
  - even/odd reflections of functions
  - diffusion equation on half-line (parabolic pdes)
    - Dirichlet case (odd reflection)
    - Neurmann case (even reflection)
  - diffusion equation on interval (parabolic pdes)
    - Dirichlet case (odd reflections)
  - wave equation on half-line (hyperbolic pdes)
    - Dirichlet case (odd reflection)
  - wave equation on interval (hyperbolic pdes)
    - Dirichlet case (odd reflections)
  - solution of inhomogeneous linear pdes (Duhamel's Principle)
    - diffusion equation with a source (parabolic pdes)
    - wave equation with a source (hyperbolic pdes)

- Separation of Variables
  - method of separation of variables
  - diffusion equation on finite interval (parabolic pdes)
    - dirichlet boundary conditions
    - neumann boundary conditions
  - wave equation on finite interval (hyperbolic pdes)
    - dirichlet boundary conditions
    - neumann boundary conditions