Name: ________________________

Final Practice Problems:

Professor: Paul J. Atzberger

Partial Differential Equations, 124A

Directions: Please answer each question carefully. Be sure to show your work. If you have questions, please feel free to ask.

Problem 1:

Use method of characteristics to find the solution of

$$\begin{cases} 4u_x(x,y) + 12x^2 u_y(x,y) = -u, & -\infty < y < \infty, x > 0 \\ u(0,y) = e^{-|y|}, & -\infty < y < \infty, x = 0 \end{cases}$$

Use method of characteristics to find the solution of

$$\begin{cases} 3u_x(x,y) + 6xu_y(x,y) = -1, & x > 0, -\infty < y < \infty \\ u(0,y) = \cos(y), & -\infty < y < \infty, x = 0 \end{cases}$$

Problem 2:

a) Use method of characteristics to find the solution of

$$\begin{cases} 6xu_x(x,y) - 3u_y(x,y) = 0, & y > 0, -\infty < x < \infty, \\ u(x,0) = \sin(x), & y = 0, -\infty < x < \infty. \end{cases}$$

b) Use method of characteristics to find the solution of

$$\begin{cases} 6u_x(x,y) + 3xu_y(x,y) = 0, & x > 0, -\infty < y < \infty, \\ u(0,y) = \cos(y), & x = 0, -\infty < y < \infty. \end{cases}$$

Problem 3:

Classify each of the following Partial Differential Equations as either elliptic, hyperbolic, or parabolic. Give a canonical form for the second-order part of each PDE.

(a) $u_{xx} - u_t + u_x = 0$.

(b) $u_{tx} + u_x + 2u = 0$.

(c) $2u_{xy} + u_{yy} + 2u_{xx} + u_y - u = 0$.

(d) $u_{xx} - u_{yy} + u_{xy} + u_x + u_y = 0$.

Problem 4:

satisfies the wave equation on the half-line

$$\begin{cases} u_{tt}(x,t) = c^2 u_{xx}(x,t), & 0 < x < \infty, t > 0 \\ u(0,t) = 0, & x = 0, t > 0 \\ u(x,0) = \phi(x), & 0 < x < \infty, t = 0 \\ u_t(x,0) = \psi(x), & 0 < x < \infty, t = 0. \end{cases}$$

a) Determine the solution to the wave equation when the initial configuration is $\phi(x) = \exp(-x)$ and initial velocity is $\psi(x) = 0$.

b) Determine the solution to the wave equation when the initial configuration is $\phi(x) = 0$ and initial velocity is $\psi(x) = \exp(-x)$.

Problem 5:

$$\begin{cases} u_{tt}(x,t) = c^2 u_{xx}(x,t) + f(x,t), & -\infty < x < \infty, t > 0 \\ u(x,0) = \phi(x), & -\infty < x < \infty, t = 0 \\ u_t(x,0) = \psi(x), & -\infty < x < \infty, t = 0. \end{cases}$$

a) Determine the solution to the wave equation when the initial configuration is $\phi(x) = 0$, the initial velocity is $\psi(x) = 0$, and the source is $f(x,t) = -1$.

b) Determine the solution to the wave equation when the initial configuration is $\phi(x) = \exp(-x)$, the initial velocity is $\psi(x) = \sin(4\pi x)$, and the source is $f(x,t) = -2$.

c) Determine the solution to the wave equation when the initial configuration is $\phi(x) = \exp(-x)$, the initial velocity is $\psi(x) = 0$, and the source is $f(x,t) = -t$.

Problem 6:

Consider the wave equation on the interval with Dirichlet Boundary Conditions

$$\begin{cases} u_{tt} = c^2 u_{xx}, & 0 < x < \ell, t > 0 \\ u(0,t) = u(\ell,t) = 0, & t > 0 \\ u(x,0) = \phi(x), & 0 < x < \ell, t = 0 \\ u_t(x,0) = \psi(x), & 0 < x < \ell, t = 0. \end{cases}$$

a) Determine the solution when $\phi(x) = \sin(2\pi x)$, $\psi(x) = 0$, and $c^2 = 2$, $\ell = 1$ using the method of Separation of Variables.

b) Determine the solution when $\phi(x) = 0$, $\psi(x) = \sin(2\pi x)$, and $c^2 = 2$, $\ell = 1$ using the method of Separation of Variables.

Problem 7:

Consider the diffusion equation on the half-line with Neumann Boundary Conditions:

$$\begin{cases} u_t = k u_{xx}, & 0 < x < \infty, t > 0 \\ u_x(0,t) = 0, & x = 0, t > 0 \\ u(x,0) = \phi(x), & 0 < x < \infty, t = 0. \end{cases}$$

a) Determine the solution when $k = 2$, $\phi(x) = \exp(-x^2)$.

Problem 8:

Consider the diffusion equation on the half-line with Dirichlet Boundary Conditions:

$$\begin{cases} u_t = k u_{xx}, & 0 < x < \infty, t > 0 \\ u(0,t) = 0, & x = 0, t > 0 \\ u(x,0) = \phi(x), & 0 < x < \infty, t = 0. \end{cases}$$

a) Determine the solution when $k = 2$, $\phi(x) = \exp(-x^2)$.

Problem 9:

Consider the diffusion equation on the real-line with a source $f(x,t)$

$$\begin{cases} u_t = ku_{xx} + f(x,t), & -\infty < x < \infty, t > 0 \\ u(x,0) = \phi(x), & -\infty < x < \infty, t = 0 \end{cases}$$

a) Determine the solution when $\phi(x) = 0$, $k = 1$, and $f(x,t) = \exp(-x^2)$.

b) Determine the solution when $\phi(x) = \exp(-x^2)$, $k = 1$, and $f(x,t) = \exp(-x^2)$.

Problem 10:

Consider the diffusion equation on the interval with Dirichlet Boundary Conditions

$$\begin{cases} u_t = \kappa u_{xx}, & 0 < x < \ell, t > 0 \\ u(0,t) = u(\ell,t) = 0, & t > 0 \\ u(x,0) = \phi(x), & 0 < x < \ell, t = 0. \end{cases}$$

a) Determine the solution when $\phi(x) = \sin(x)$, $\ell = 2\pi$, and $k = 2$ by using the method of Separation of Variables.