



Midterm Exam Outline

Math 124A: Numerical Analysis

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- First-Order PDEs and Method of Characteristics
 - constant coefficient case
 - form of the characteristics (lines)
 - general solutions
 - variable coefficient case when $a(x, y) > 0$
 - characteristic equations
 - form of the characteristics based on $y(x)$
 - general solutions
 - general variable coefficient case
 - characteristic equations for $y(s)$
 - general solutions
 - examples
 - transport PDE
- Second-Order PDEs
 - linear change-of-variables
 - relationship between the gradients in each coordinate system
 - how second-order differential operators transform under coordinate changes
 - classifying second-order PDEs
 - expressing PDE similar to a quadratic form in matrix-vector notation
 - elliptic case
 - conditions on the coefficients a_{ij}
 - canonical form when first-order terms are zero
 - hyperbolic case
 - conditions on the coefficients a_{ij}
 - canonical form when first-order terms are zero
 - parabolic case
 - conditions on the coefficients a_{ij}
 - canonical form when all but one first-order term is zero
 - definition of well-posedness.
 - sufficient criteria for well-posedness.
 - uniqueness, existence, robustness to perturbations.
- Hyperbolic PDEs
 - wave equation initial value problem
 - solution technique in 1D on \mathbb{R}^1
 - form of the general solution
 - example

- hat function initial conditions
- domain of dependence
- domain of influence
- kinetic and potential energy of solutions
- conservation of energy principle

- Parabolic PDEs
 - diffusion equation initial value problem
 - uniqueness
 - stability
 - solution technique in 1D on R^1
 - properties of the diffusion equation
 - special solution (Green's function)
 - form of the general solution
 - entropy production
 - maximum principle