

Representations of Braid Groups

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Abstract

In this paper we survey some work on representations of B_n given by the induced action on a homology module of some space. One of these, called the Lawrence-Krammer representation, recently came to prominence when it was shown to be faithful for all n . We will outline the methods used, applying them to a closely related representation for which the proof is slightly easier. The main tool is the Blanchfield pairing, a sesquilinear pairing between elements of relative homology. We discuss two other applications of the Blanchfield pairing, namely a proof that the Burau representation is not faithful for large n , and a homological definition of the Jones polynomial. Finally, we discuss possible applications to the representation theory of the Hecke algebra, and ultimately of the symmetric group over fields of non-zero characteristic.

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1. Introduction

Artin's braid group B_n was originally defined as a group of geometric braids in \mathbf{R}^3 . Representations of B_n have been studied for their own intrinsic interest, and also in connection to other areas of mathematics, most notably to knot invariants such as the Jones polynomial.

We will use the definition of B_n as the mapping class group of an n -times punctured disk D_n . A rich source of representations of B_n is the induced action on homology modules of spaces related to D_n . The Burau representation, one of the simplest and best known representations of braid groups, is most naturally defined as the induced action of B_n on the first homology of a cyclic covering space of D_n . Lawrence [9] extended this idea to configuration spaces in D_n , and was able to obtain all of the so-called Temperley-Lieb representations.

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Lawrence's work seems to have received very little attention until one of her homological representations was shown to be faithful, thus proving that braid groups are linear and solving a longstanding open problem. Two independent and very different proofs of this have appeared in [1] and [8]. In this paper we will outline the former, emphasising the importance of the Blanchfield pairing. We then discuss two other applications of the Blanchfield pairing, namely the proof that the Burau representation is not faithful for large n , and a homological definition of the Jones polynomial of a knot. We conclude with some speculation on possible future applications to representations of Hecke algebras when q is a root of unity. These are related to representations of the symmetric group S_n over fields of bad characteristic, that is, fields in which $n! = 0$.

2. The Lawrence-Krammer representation

Let D be the unit disk centred at the origin in the complex plane. Fix arbitrary real numbers $-1 < p_1 < \dots < p_n < 1$, which we will call "puncture points". Let

$$D_n = D \setminus \{p_1, \dots, p_n\}$$

be the n -times punctured disk. The braid group B_n is the mapping class group of D_n , that is, the set of homeomorphisms from D_n to itself that act as the identity on ∂D , taken up to isotopy relative to ∂D . Let C be the space of all unordered pairs of distinct points in D_n .

Suppose x is a point in D_n , and α is a simple closed curve in D_n enclosing one puncture point and not enclosing x . Let $\gamma: I \rightarrow C$ be the loop in C given by

$$\gamma(s) = \{x, \alpha(s)\}.$$

Now suppose τ_1 and τ_2 are paths in D_n such that $\tau_1\tau_2$ is a simple closed curve that does not enclose any puncture points. Let $\tau: I \rightarrow C$ be the loop in C given by

$$\tau(s) = \{\tau_1(s), \tau_2(s)\}.$$

Let

$$\Phi: \pi_1(C) \rightarrow \langle q \rangle \oplus \langle t \rangle$$

be the unique homomorphism such that $\Phi(\gamma) = q$ and $\Phi(\tau) = t$ for any γ and τ defined as above. For a proof of the existence and uniqueness of such a homomorphism, see [11]. Let \tilde{C} be the connected covering space of C whose fundamental group is the kernel of Φ .

The second homology $H_2(\tilde{C})$ is a module over $\mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$, where q and t act by covering transformations. The Lawrence-Krammer representation of B_n is the induced action of B_n on $H_2(\tilde{C})$ by $\mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$ -module automorphisms. More precisely, given an element of B_n represented by a homeomorphism $\sigma: D_n \rightarrow D_n$, consider the induced action $\sigma: C \rightarrow C$. There is a unique lift $\tilde{\sigma}: \tilde{C} \rightarrow \tilde{C}$ that acts as the identity on $\partial \tilde{C}$. This induces an automorphism of $H_2(\tilde{C})$, which can be shown to respect the $\mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$ -module structure. See [11] for details.

3. The Blanchfield pairing

Let $\epsilon > 0$ be small. We define $P, B \subset C$ as follows. Suppose $\{x, y\}$ is a point in C . We say $\{x, y\} \in P$ if either $|x - y| \leq \epsilon$, or there is a puncture point p_i such that $|x - p_i| \leq \epsilon$ or $|y - p_i| \leq \epsilon$. We say $\{x, y\} \in B$ if $x \in \partial D$ or $y \in \partial D$.

For $u \in H_2(\tilde{C}, \tilde{P})$ and $v \in H_2(\tilde{C}, \tilde{B})$ let $\langle u \cdot v \rangle \in \mathbf{Z}$ denote the standard algebraic intersection number. We define an intersection pairing

$$\langle \cdot, \cdot \rangle: H_2(\tilde{C}, \tilde{P}) \times H_2(\tilde{C}, \tilde{B}) \rightarrow \mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$$

by

$$\langle u, v \rangle = \sum_{i, j \in \mathbf{Z}} (u \cdot q^i t^j v) q^i t^j.$$

For a proof that these are well-defined, see [7, Appendix E], where the following properties are also proved.

For $u \in H_2(\tilde{C}, \tilde{P})$, $v \in H_2(\tilde{C}, \tilde{B})$, $\sigma \in B_n$, and $\lambda \in \mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$, we have

$$\langle \sigma u, \sigma v \rangle = \langle u, v \rangle,$$

and

$$\langle \lambda u, v \rangle = \lambda \langle u, v \rangle = \langle u, \bar{\lambda} v \rangle,$$

where $\bar{\lambda}$ is the image of λ under the automorphism of $\mathbf{Z}[q^{\pm 1}, t^{\pm 1}]$ taking q to q^{-1} and t to t^{-1} .

4. A faithful representation

The aim of this section is to outline a proof of the following.

Theorem. *Let \tilde{C} and \tilde{P} be as above. The induced action of B_n on $H_2(\tilde{C}, \tilde{P})$ is faithful.*

For the details, the reader is referred to [1], where the same techniques are used to prove that B_n acts faithfully on $H_2(\tilde{C})$. Our use of relative homology here actually simplifies the argument somewhat.

There is a slight technical difficulty in defining the action of B_n on $H_2(\tilde{C}, \tilde{P})$. Namely, the action of a braid on C need not preserve the set P . Thus we should really take a limit as ϵ approaches 0. The representation obtained is very similar to the Lawrence-Krammer representation, but has a slightly different module structure, as discussed in [3].

Let E be the straight edge from p_1 to p_2 . Let E' be the set of points in C of the form $\{x, y\}$, where $x, y \in E$. Let \tilde{E}' be a lift of E' to \tilde{C} . This represents an element of $H_2(\tilde{C}, \tilde{P})$, which we will call e . Let F_1 and F_2 be parallel vertical edges with endpoints on ∂D , passing between p_2 and p_3 . Let F' be the set of points in C of the form $\{x, y\}$, where $x \in F_1$ and $y \in F_2$. Let \tilde{F}' be a lift of F' to \tilde{C} . This represents an element of $H_2(\tilde{C}, \tilde{B})$, which we will call f . Note that

$$\langle e, f \rangle = 0,$$

since E' and F' are disjoint in C .

Suppose the action of B_n on $H_2(\tilde{C}, \tilde{F})$ is not faithful. It is not hard to show that there must be a braid σ in the kernel of this representation such that $\sigma(E)$ is not isotopic to E relative to endpoints. Now $\sigma(e) = e$, so

$$\langle \sigma(e), f \rangle = 0.$$

From this, we will derive a contradiction.

By applying an isotopy, we can assume $\sigma(E)$ intersects F_1 and F_2 transversely at a minimal number of points $k > 0$. Let x_1, \dots, x_k be the points of $\sigma(E) \cap F_1$, and let y_1, \dots, y_k be the points of $\sigma(E) \cap F_2$, numbered from top to bottom in both cases.

For $i, j \in \{1, \dots, k\}$, let $a_{i,j}$ and $b_{i,j}$ be the unique integers such that $\sigma(\tilde{E}')$ intersects $q^{a_{i,j}} t^{b_{i,j}} \tilde{F}'$ at a point in the fibre over $\{x_i, y_j\}$, and let $\epsilon_{i,j}$ be the sign of that intersection. Then

$$\langle \sigma(e), f \rangle = \sum_{i=1}^k \sum_{j=1}^k \epsilon_{i,j} q^{a_{i,j}} t^{b_{i,j}}.$$

To calculate $a_{i,j}$ and $b_{i,j}$, it is necessary to specify choices of lift for E' and F' . We will not do this since we only need to calculate differences $a_{i',j'} - a_{i,j}$ and $b_{i',j'} - b_{i,j}$. To do this, let γ be a path in C that goes from $\{x_i, y_j\}$ to $\{x_{i'}, y_{j'}\}$ in $\sigma(E')$, and then back to $\{x_i, y_j\}$ in F' . Then

$$q^{(a_{i',j'} - a_{i,j})} t^{(b_{i',j'} - b_{i,j})} = \Phi(\gamma).$$

From this we can prove the following.

Lemma. For all $i, j \in \{1, \dots, k\}$ we have

- $a_{i,j} = \frac{1}{2}(a_{i,i} + a_{j,j})$,
- if $b_{i,j} > b_{i,i}$ then $a_{i',j'} > a_{i,i}$ for some $j' = 1, \dots, k$,
- if $b_{i,j} > b_{j,j}$ then $a_{i',j'} > a_{j,j}$ for some $i' = 1, \dots, k$.

The first of these is [1, Lemma 2.1], and the second and third follow from the proof of [1, Claim 3.4]. We now sketch the proof of the second in the special case where y_i lies between x_i and y_j along $\sigma(E)$.

Let α be the path from y_i to y_j along $\sigma(E)$. Let β be the path from y_j to y_i along F_2 . Then $b_{i,j} - b_{i,i}$ is two times the winding number of $\alpha\beta$ around x_i . In particular, this winding number is positive.

Let D_1 be the once punctured disk $D \setminus \{x_i\}$, and let \tilde{D}_1 be its universal cover. Let $\tilde{\alpha}\tilde{\beta}$ be a lift of $\alpha\beta$ to D_1 . This is a path from a point in the fibre over y_i to a "higher" point in the fibre over y_i .

Let F_2^+ be the the segment of F_2 going from y_i upwards to ∂D . Let \tilde{F}_2^+ be the lift of F_2^+ to \tilde{D}_1 that has an endpoint at $\tilde{\alpha}(0)$. In order to reach a higher sheet in \tilde{D}_1 , $\tilde{\alpha}$ must intersect \tilde{F}_2^+ . Let $\tilde{\gamma}$ be the loop in \tilde{D}_1 that follows $\tilde{\alpha}$ to the first point of intersection with \tilde{F}_2^+ , and then follows \tilde{F}_2^+ back to $\tilde{\alpha}(0)$.

Let γ be the projection of $\tilde{\gamma}$ to D_1 . This travels along $\sigma(E)$ from y_i to some point $y_{j'} \in F_2$, then along F_2 back to y_i . Then $a_{i,j'} - a_{i,i}$ is the total winding

number of γ around the puncture points. We must show that this winding number is positive.

By construction, $\tilde{\gamma}$ is a simple closed curve in \tilde{D}_1 . By the Jordan curve theorem, it must bound a disk \tilde{B} . Let B be the projection of \tilde{B} to D_1 . This is an immersed disk in D , whose boundary is γ . Note that $\tilde{\gamma}$ passes anticlockwise around \tilde{B} , since the puncture x_i lies to its right. Thus the total winding number of γ around the puncture points is equal to the total number of puncture points contained in B , counted with multiplicities.

It remains to show that B intersects at least one puncture point. Suppose not. Then B is an immersed disk in D_n . Using an "innermost disk" argument, one can find an embedded disk B' in D_n whose boundary consists of a subarc of $\sigma(E)$ and a subarc of F_2 . Using B' , one can isotope $\sigma(E)$ so as to have fewer points of intersection with F_2 , thus contradicting our assumptions.

This completes the proof of the second part of the lemma in the case where y_i lies between x_i and y_j along $\sigma(E)$. The remaining case, where x_i lies between y_i and y_j , is only slightly trickier. The third part of the lemma is similar to the second. The first part of the lemma is much easier.

We now return to the proof of the theorem. Let a be the maximum of all $a_{i,j}$. Let b be the maximum of $\{b_{i,j} : a_{i,j} = a\}$. Suppose $i, j \in \{1, \dots, k\}$ are such that $a_{i,j} = a$ and $b_{i,j} = b$, and also $i', j' \in \{1, \dots, k\}$ are such that $a_{i',j'} = a$ and $b_{i',j'} = b$. I claim that $\epsilon_{i,j} = \epsilon_{i',j'}$. From this claim, it follows that all occurrences of $q^a t^b$ in the expression

$$\langle \sigma(e), f \rangle = \sum_{i=1}^k \sum_{j=1}^k \epsilon_{i,j} q^{a_{i,j}} t^{b_{i,j}}$$

occur with the same sign, so the coefficient of $q^a t^b$ is non-zero in $\langle \sigma(e), f \rangle$. This provides our desired contradiction, and completes the proof of the theorem. It remains to prove that $\epsilon_{i',j'} = \epsilon_{i,j}$.

Using the above lemma, it is not hard to show that $a_{i,i} = a_{j,j} = a$ and $b_{i,i} = b_{j,j} = b$. Similarly, $a_{i',i'} = a_{j',j'} = a$ and $b_{i',i'} = b_{j',j'} = b$. We will only need the equalities

$$b_{i,i} = b_{i,j} = b_{j,j} = b_{i',i'} = b_{i',j'} = b_{j',j'}.$$

In fact, we only need these modulo two.

Orient $\sigma(E)$ so that it crosses F_1 from left to right at x_i . Let γ be the path in C which goes from $\{x_i, y_i\}$ to $\{x_{i'}, y_{i'}\}$ in E' and then back to $\{x_i, y_i\}$ in F' . Now $b_{i',i'} - b_{i,i}$ is the exponent of t in $\Phi(\gamma)$. The fact that this is an even number means that the pair of points in D_n do not "switch places" when they go around this loop. Thus $\sigma(E)$ crosses F_1 from left to right at $x_{i'}$. By similar arguments,

- $\beta(E)$ intersects F_1 with the same sign at x_i and $x_{i'}$,
- $\beta(E)$ intersects F_2 with the same sign at y_j and $y_{j'}$,
- x_i occurs before y_j and $x_{i'}$ occurs before $y_{j'}$ with respect to the orientation of $\sigma(E)$.

It is now intuitively clear that E' must intersect F' with the same signs at $\{x_i, y_j\}$ and $\{x_{i'}, y_{j'}\}$. This can be proved rigorously by careful consideration of orientations

of these surfaces, as discussed in [1, Section 2.1]. It follows that $\epsilon_{i,j} = \epsilon_{i',j'}$, which completes the proof of the theorem.

5. The Burau representation

The proof that the Lawrence-Krammer representation is faithful basically reduces to proving that the Blanchfield pairing detects whether corresponding edges in the disk can be isotoped to be disjoint. A converse to this idea leads to a proof that the Burau representation is not faithful for large n .

The Burau representation can be defined by a similar but simpler construction to that of the Lawrence-Krammer representation. Let

$$\Phi: \pi_1(D_n) \rightarrow \langle q \rangle$$

be the homomorphism that sends each of the obvious generators to q . Let \tilde{D}_n be the corresponding covering space. The Burau representation is the induced action of B_n on $H_1(\tilde{D}_n)$ by $\mathbf{Z}[q^{\pm 1}]$ -module automorphisms.

Let P be an ϵ -neighbourhood of the puncture points, and let \tilde{P} be the preimage of P in \tilde{D}_n . The Blanchfield pairing in this context is a sesquilinear pairing

$$\langle \cdot, \cdot \rangle: H_1(\tilde{D}_n, \tilde{P}) \times H_1(\tilde{D}_n, \partial \tilde{D}_n) \rightarrow \mathbf{Z}[q^{\pm 1}].$$

Let E be the straight edge from p_1 to p_2 . Let \tilde{E} be a lift of E to \tilde{D}_n . This represents an element of $H_1(\tilde{D}_n, \tilde{P})$, which we will call e . Let F be a vertical edge with endpoints on ∂D , passing between p_{n-1} and p_n . Let \tilde{F} be a lift of F to \tilde{D}_n . This represents an element of $H_1(\tilde{D}_n, \partial \tilde{D}_n)$, which we will call f . The following is [2, Theorem 5.1].

Theorem. *Let E, e, F and f be as above. The Burau representation of B_n is unfaithful if and only if there exists $\sigma \in B_n$ such that $\langle \sigma(e), f \rangle = 0$, but $\sigma(E)$ is not isotopic relative to endpoints to an edge that is disjoint from F .*

Using this theorem, one can show that the Burau representation of B_n is not faithful simply by exhibiting the required edges $\sigma(E)$ and F . Such edges can be found by hand in the case $n = 6$. In the case $n = 5$, they can be found by a computer search, and then laboriously checked by hand. The case $n = 4$ seems to be beyond the reach of any known computer algorithm. This is the last open case, since the Burau representation is known to be faithful for $n \leq 3$.

6. The Jones polynomial

In this section, we use the Blanchfield pairing to give a homological definition of the Jones polynomial of a knot or link. The Jones polynomial was defined in [6] using certain algebraically defined representations of braid groups. No satisfactory geometric definition is known, but some insight might be offered by defining the representations homologically and using the Blanchfield pairing. This was the original motivation for Lawrence to study homological representations of braid groups.

A *geometric braid* $\sigma \in B_n$ is a collection of n disjoint edges in $\mathbf{C} \times \mathbf{R}$ with endpoints $\{1, \dots, n\} \times \{0, 1\}$, such that each edge goes from $\mathbf{C} \times \{0\}$ to $\mathbf{C} \times \{1\}$ with a constantly increasing \mathbf{R} component. The correspondence between geometric braids and elements of the mapping class group is described in [1], and in many other introductory expositions on braids. The *plat closure* of a geometric braid $\sigma \in B_{2n}$ is the knot or link obtained by using straight edges to connect $(2j - 1, k)$ to $(2j, k)$ for each $j = 1, \dots, n$ and $k = 0, 1$. Every knot or link can be obtained in this way.

Let C be the configuration space of unordered n -tuples of distinct points in D_{2n} . Let

$$\Phi: \pi_1(C) \rightarrow \langle q \rangle \oplus \langle t \rangle$$

be defined as in Section 2. Namely, if γ is any loop in which one of the n -tuple winds anticlockwise around a puncture point, and τ is any loop in which two of the n -tuple exchange places by an anticlockwise half twist, then $\Phi(\gamma) = q$ and $\Phi(\tau) = t$. Let \tilde{C} be the covering space corresponding to Φ .

Define $P, B \subset C$ similarly to those of Section 3. The Blanchfield pairing is a sesquilinear pairing

$$\langle \cdot, \cdot \rangle: H_2(\tilde{C}, \tilde{P}) \times H_2(\tilde{C}, \tilde{B}) \rightarrow \mathbf{Z}[q^{\pm 1}, t^{\pm 1}].$$

For $k = 1, \dots, n$, let F_k be the straight edge from p_{2k-1} to p_{2k} . Let F be the set of points in C of the form $\{x_1, \dots, x_n\}$ where $x_i \in F_i$. Let \tilde{F} be a lift of F to \tilde{C} . This represents an element of $H_n(\tilde{C}, \tilde{P})$, which we call f . For $k = 1, \dots, n$, let $e_k: S^1 \rightarrow D_n$ be a figure-eight around p_{2k-1} and p_{2k} in a small neighbourhood of F_k . Let e be the map from the n -torus $(S^1)^n$ to C given by

$$e(s_1, \dots, s_n) = \{e_1(s_1), \dots, e_n(s_n)\}.$$

Let \tilde{e} be a lift of e to \tilde{C} . This represents an element of $H_n(\tilde{C})$, and hence of $H_n(\tilde{C}, \tilde{B})$, which we also call e .

The main result of [4] is the following.

Theorem. *Let e and f be as above and suppose $\sigma \in B_{2n}$. The Jones polynomial of the plat closure of σ is*

$$\langle \sigma(e), f \rangle |_{(t=-q^{-1})},$$

up to sign and multiplication by a power of $q^{\frac{1}{2}}$.

Here, the Jones polynomial is normalised so that the Jones polynomial of the unknot is $-q^{\frac{1}{2}} - q^{-\frac{1}{2}}$. The correct sign and power of $q^{\frac{1}{2}}$ is also explicitly specified in [4].

This result is due to Lawrence, who also achieved a similar result for the two-variable Jones polynomial by a much more complicated construction [10].

7. The Hecke algebra

We conclude with some speculation about possible applications of the Blanchfield pairing to the representation theory of Hecke algebras. We first give a very brief overview of the basic theory of Hecke algebras.

Let $q \in \mathbf{C} \setminus \{0\}$. The Hecke algebra $H_n(q)$, or simply H_n , is the \mathbf{C} -algebra given by generators g_1, \dots, g_{n-1} and relations

- $g_i g_j = g_j g_i$ if $|i - j| > 1$,
- $g_i g_j g_i = g_i g_j g_i$ if $|i - j| = 1$,
- $(g_i - 1)(g_i + q) = 0$.

It is an $n!$ -dimensional \mathbf{C} -algebra. We are restricting to the ring \mathbf{C} for convenience, although other rings can be used.

Note that $H_n(1)$ is the group algebra $\mathbf{C}S_n$ of the symmetric group S_n . The Hecke algebra is called a "quantum deformation" of $\mathbf{C}S_n$. The representation theory of $\mathbf{C}S_n$ is well understood except when working over a field of finite characteristic in which $n! = 0$. This is because the classical theory sometimes requires one to divide by $n!$, the order of the group. When studying H_n it turns out to be useful to be able to divide by

$$(1 + q + \dots + q^{n-1})(1 + q + \dots + q^{n-2}) \dots (1 + q).$$

This is sometimes written as $[n]!$, and can be thought of as a "quantum deformation" of $n!$. Note that $[n]! = n!$ if $q = 1$. A *generic* value of q is one for which $[n]! \neq 0$. The non-generic values are the primitive k th roots of unity for $k = 2, \dots, n$. The representation theory of H_n is well understood for generic values of q , but the non-generic values are the subject of ongoing active research.

One of the most important papers on this subject is Dipper and James [5]. For every partition λ of n , they define a H_n -module S^λ called the *Specht module*. They then define a bilinear pairing on S^λ , which we denote $\langle \cdot, \cdot \rangle_{\text{DJ}}$. Let S^λ_\perp denote the set of $u \in S^\lambda$ such that $\langle u, v \rangle_{\text{DJ}} = 0$ for all $v \in S^\lambda$. Let D^λ be the quotient module $S^\lambda / S^\lambda_\perp$. Dipper and James show that the non-zero D^λ form a complete list of all distinct irreducible representations of H_n . For generic values of q we have $D^\lambda = S^\lambda$. For non-generic values of q , the D^λ are not well understood.

Lawrence [10] gave a homological definition of the Specht modules. The construction begins with the action of B_n on a homology module of a configuration space. The variable t is then specialised to $-q^{-1}$, and a certain quotient module is taken. A detailed treatment of the case $\lambda = (n-2, 2)$ is given in [3].

There is a Blanchfield pairing on the Specht modules as defined by Lawrence. It would be nice if this were the same as the pairing defined by Dipper and James. Unfortunately the Blanchfield pairing is sesquilinear, whereas the pairing defined by Dipper and James is bilinear. This problem can be overcome as follows. Let $\rho: D_n \rightarrow D_n$ be the conjugation map. Let $\tilde{\rho}$ be the induced map on $H_k(\tilde{C}, \tilde{B})$. Then the pairing

$$\langle u, v \rangle' = \langle u, \rho(v) \rangle$$

can be shown to give a bilinear pairing on the Specht module.

For generic values of q , this topologically defined pairing is the same as the algebraically defined pairing of Dipper and James, up to some renormalisation.

There is some evidence that this can be made to work at non-generic values of q . If so, it would give rise to a new homological definition of the modules D^λ , and new topological tools for studying them. In any case, it would be interesting to better understand the behaviour of this Blanchfield pairing at roots of unity.

References

- [1] S. Bigelow, Braid groups are linear, *J. Amer. Math. Soc.*, 14 (2001), 471–486.
- [2] S. Bigelow, Does the Jones polynomial detect the unknot? *J. Knot Theory Ramifications*, (to appear).
- [3] S. Bigelow, The Lawrence-Krammer representation, *Proceedings, Georgia Topology Conference, 2001*, (to appear).
- [4] S. Bigelow, A homological definition of the Jones polynomial, *Proceedings, RIMS, Kyoto, 2001*, (to appear).
- [5] R. Dipper & G. James, Representations of Hecke algebras of general linear groups, *Proc. London Math. Soc. (3)*, 52 (1986), 20–52.
- [6] V. Jones, A polynomial invariant for knots via von Neumann algebras, *Bull. Amer. Math. Soc. (N.S.)*, 12 (1985), 103–111.
- [7] A. Kawachi, *A survey of knot theory*, Birkhäuser Verlag, 1996.
- [8] D. Krammer, Braid groups are linear, *Ann. of Math. (2)*, 155 (2002), 131–156.
- [9] R. Lawrence, Homological representations of the Hecke algebra, *Comm. Math. Phys.*, 135 (1990), 141–191.
- [10] R. Lawrence, Braid group representations associated with sl_m , *J. Knot Theory Ramifications*, 5 (1996), 637–660.
- [11] L. Paoluzzi & L. Paris, A note on the Lawrence-Krammer-Bigelow representation, *Algebr. Geom. Topol.*, 2 (2002), 499–518.