MATHEMATICS 3C
Introduction to Differential Equations and Linear Algebra
Summer Session B 2007

Instructor: Brie Finegold, briefly@math.ucsb.edu, 6432H South Hall

Office Hours: Monday, Wednesday, Thursday 11:00--12:00 AM or by appointment
Class Meets 9:30-10:40, 12:30-1:40 MTWRF

Text: *Differential Equations and Linear Algebra* by Farlow, Hall, McDill, and West

Overview of the Course: We will focus on recognizing applications of and solving first order differential equations. Linear Algebra, a tool for solving more complicated differential equations, such as those in 5A, will also be a major topic of the course. Problem-solving skills and the subject matter of the class will be developed simultaneously via in-class collaboration and independent done work outside of class. There will be a few short lectures. The course will require considerable out-of-class work, including solving problems from the text and preparing for in-class participation. Assessment will be based on both in-class and out-of-class assignments.

The course grade will be based on the following:

Attendance	5 %
Problem Sets/Assignments/Interviews	20 %
Group participation, presentations and write-ups	20 %
Midterms (15% each)	30 %
Final (In-class and Take-home parts)	25 %

Attendance: *Consistent attendance is highly recommended.* Attendance counts towards your grade and group activity will be a regular part of this class. There are unforeseen emergencies that do come up. However, check your schedule ahead of time and plan on missing no more than two classes. There is no make-up of missed group activity.

Math Lab: South Hall 1607, open Monday through Friday, 12:00 - 5:00 p.m. A Teaching Assistant will be available to assist you with mathematical questions you may have.

Office Hours: Please take advantage of my office hours. I actually look forward to students dropping by and discussing mathematics. Don't be shy in making regular appearances.

Online Resources:

- http://www.math.lsa.umich.edu/courses/116/slopefields.html is the address of the slope field applet we will use.
- www.mathnerds.org is a homework help resource. This site gives helpful hints to get you started but will not give you "the answer".
- www.facebook.com is a networking system that we will use. Join the Facebook group "3C Session B with Brie" to get up-to-date info on the class and to help you find study partners for the class.

Pedagogy: My role will be that of a facilitator to *our* construction of mathematical ideas

during the term. You may find that this class is taught much differently than the typical lecture format of many mathematics classes--though there will be some lectures interspersed during the term. This is an important shift in teaching practice for many of us. Often, portions of our class time will be spent working in groups in an effort to develop solution strategies for various problems and explorations.

Overall Philosophy: I require written explanations on all homework, group investigations, and exams. I hope to avoid the "turn the crank" style of computation that is typical of many mathematics courses. Solution of homework problems will require careful thinking. You will not always be able to solve the problems by imitating a procedure found in class or in the book. **Warning!** The answers you may find in "solutions or answer books" often represent **only partial solutions**. These are usually not sufficient to receive complete credit.
Also: Just getting an answer is not enough; you are expected to explain connections between ideas and think about extensions of you work. You may wish to use answers as hints for certain problems, but they are not models for complete solutions. If you are stuck, work with a classmate, go to the Math Lab, send me e-mail, and/or bring questions to class meetings. On all assignments and exams, it is crucial that you explain, in complete sentences, what you are thinking. It is possible to receive a poor score for a correct answer if you do not communicate to me your ideas. On the other hand, a clear exposition with a minor computational error can receive a good score.

Homework/Interviews: During 3C, you may be asked to do some explorations using the computer. A working knowledge of Microsoft Excel will suffice for these projects. Homework from the book will be assigned and periodically collected to check one or two problems at random. I will sometimes write up model solutions to which you may compare your work. You will be required to set up three interviews with me during the session so that we can discuss your progress and you can get feedback.

A Final Comment: I want this to be a successful and enjoyable learning experience for you. During the term, I hope that you will reflect carefully on any plans you might have for a future career related to mathematics. Please feel free to contact me if you have questions. Using e-mail is probably the most efficient way of reaching me. It's also a way of getting clarification on homework problems.

Math 3C Differential Equations and Linear Algebra

Date **Tentative Discussion Topics &Class Explorations**

Part 1: Techniques for solving first order differential equations

08/06 Course expectations/Definitions
In-class Exploration: Modeling: spread of disease – non-linear system, predator-prey – linear system, falling bodies – 2^{nd} order, population growth –1^{st} order

- **Answer questions from 3C Refresher handout**
- **Finish Modeling handout**

08/07 Mini-lecture: Direction Fields --- stable/unstable, equilibria, constant, long-term, concavity
In-class Exploration: plotting and interpreting direction fields using transparencies

- **pages 20-24 #'s 2,6,14, 16-21, 23, 26, 29, 30**
- **Start Assignment 1 (ignore parts about Euler method)**
- **Determine which equations from Assignment 1 are autonomous (refer to top of page 24)**

08/08 Mini-lecture: The definition of derivative as it applies to D.E.'s
In-class Exploration: Numerical methods for solving D.E.'s **set up first interview**

- **Finish Assignment 1 (Do the Euler Method part)**

08/09 Student-led discussion of numerical results and direction fields **Assignment 1 DUE**
In-class exploration: calculating e with Euler's method

- **page 43 #'s 13, 14**
- **page 45 #29 – remind to go over in class**
- **pages 29-31 #'s 48, 49, 50**

08/10 Mini-lecture: The Fundamental Theorem of Calculus as it applies to D.E.'s
In-class Exploration: integration and non-numerical solutions for first order homogeneous DE's

- **pages 29-31 #'s 21, 24, 41,42**
- **Use integrating methods to solve as many of the D.E.'s from assignment one as possible**

08/13 Student-led discussion of Problem Set 1
In-class Exploration: Linearity/Composition of Functions **Problem Set 2 DUE**

- **page 62 #'s 2,4,6,8,10, 25, 34, 38, 44**

08/14 In-class Exploration: Homogeneous/Non-homogeneous equations and their relationship

- **page 70 #'s 2, 4, 6, 8, 10**

08/15 The Product Rule as it applies to D.E.'s
Exploration: Another non-numerical method for solving D.E.'s

- **page 70 #'s 22, 24, 26, 33, 34**
- **Read Picard' Theorem on page 49 and look at example 3**

08/16 Short discussion of Picard's Theorem
Student-led discussion of Problem Set 3 and Review **Problem Set 3 Due**

08/17 **First Midterm**

Part 2: Applications of first order differential equations

08/20 Mini-lecture: connections between pre-midterm/post mid-term material
Exploration: Growth and Decay (Biology)

08/21 Exploration: Logistic Equation (Biology)

08/22 Exploration: Mixing/Cooling (Chemistry) **Problem Set 3 due/ set up 2nd interview**

08/23 Exploration: Free-fall or periodic motion (Physics)

08/24 Lecture on Systems of Differential Equations and the usefulness of Linear Algebra

08/27 Exploration/Demonstration Predator Prey Model **Problem Set 4 Due**

08/28 Composition of functions and chain rule
Exploration: Introduction to Matrices

08/29 Systems of Equations rewritten as matrix equations
Exploration: patterns in Matrix multiplication **Short Problem Set 5 Due**

08/31 **Second Midterm**

09/03 Labor Day/No Class

Part 3: Intro to Matrices and Linear Algebra

09/04 Exploration: area of a parallelogram and determinant **set up 3rd interview**

09/05 Exploration: Gaussian elimination

09/06 Exploration: A formula for the inverse of a 2X2 matrix

09/07 Lecture: Vector spaces **Problem Set 6 Due**

09/10 Exploration: linear independence of polynomials

09/11 Exploration: Wronskian **Take home portion of final assigned**

09/12 Lecture: Basis and dimension

09/13 Review

09/14 Final

Assignment 1: This assignment is meant to help you understand how solutions to differential equations can be sketched using a computer. You will plot direction fields and discuss behavior of your group's differential equation.

- NOTE: k and r stand for constants. First vary the constants to produce at least 3 different equations (don't forget to try negative numbers and 0).
- Plot their direction fields and families of solutions using the applet.
- Answer the questions at the top of page 23 about long-term behavior.
- For one of your solutions demonstrate a calculation that shows how Euler's method works for the first two iterations (check the calculation by making sure it matches the plot on the applet).
- Explain how the nature of the solutions changes as the constants change.
- Experiment with changing the step size of Euler's method. Comment on the accuracy of Euler's method, and try to find a differential equation that "breaks" the applet—i.e. produces a poor looking answer.
- Explain why this equation is not solved well by Euler's method. Notice the different choices there are for methods on the applet, and see which method produces the best-looking answer.

Prepare to discuss your observations in class by discussing them amongst your group.

i) **p'(x)=rp(x)(k-p(x))**
ii) **p'(x)p(x)= k**
iii) **(p'(x))^2=kp(x)**
iv) **rp'(x)+kp(x)= cosx**
v) **rp'(x)+kp(x)=x^2**
vi) **rp'(x)+kp(x)=5**
vii) **rp'(x)+kxp(x)=5**
viii) **rp'(x)+ksinxp(x)=5**
ix) **p'(x)=p(x)^2 + k**
x) **p'(x)=1/(x+kp(x))**
xi) **p'(x)=sin(kp(x))**
xii) **p'(x)p(x)=kx**
xiii) **p'(x)x = kp(x)**

Problem Set 2: All problems assigned between 08/06 and 08/09

Problem Set 3: All problems assigned between 08/10 and 08/15

Problem Set 4:

Problem Set 5:

Problem Set 6: