Linear Functions

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Let's think about composing linear functions. Suppose n, m are positive integers. A function $L : \mathbb{R}^n \to \mathbb{R}^m$ is **linear** means that for each pair of vectors, $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$ in $\mathbb{R}^n$, $L(x + y) = L(x) + L(y)$.

The goal is to find an efficient means of representing and composing linear functions.

1. Give an example of a linear function with:
   (a) domain $\mathbb{R}^2$ and range $\mathbb{R}^2$,
   (b) domain $\mathbb{R}^2$ and range $\mathbb{R}^3$
   (c) domain $\mathbb{R}^3$ and range $\mathbb{R}^3$
   (d) domain $\mathbb{R}^3$ and range $\mathbb{R}^2$
   (e) domain $\mathbb{R}^1$ and range $\mathbb{R}^2$
   (f) domain $\mathbb{R}^3$ and range $\mathbb{R}^1$

   Look for patterns amongst the examples of each type of function found by members of your group. What is the general form taken by a linear function?

2. Can you compose any two of the six functions listed above? Explain.

3. Compose several pairs of functions from your list, and discuss patterns you see.

4. I am thinking of a linear function, L, with domain and range $\mathbb{R}^2$ with the property that $L(1, 0) = (2, 1)$ and $L(0, 1) = (3, 2)$. What is $L(x_1, x_2)$? Justify your answer.

5. Use the methods established in class to quickly compose $g(x_1, x_2) = (x_1, x_2, x_2 + x_2)$ and $f(y_1, y_2, y_3) = y_1 + y_2 + 2y_3$.

6. Write a paragraph explaining the relationship between matrices and linear functions to a college freshman who hasn't taken 3C.