Math 3C: Modeling August 06,2007

S-I-R Model for spread of disease and discrete modeling A small Island has population $50,000$. An infectious disease X has been spreading for some time and your task is to predict its future course. Disease X is not fatal provided the patient's fever is kept down, and full recovery with immunity results in 14 days after infection. Once recovered, the person is no longer infectious. According to your statistics, 2100 people are currently infected and 2500 have recovered. A recent article in the New Guinea Journal of Medicine says that the rate of new infections for disease X can be calculated as $10^{-5}SI$ new infections per day, where S represents the number of people in the who are susceptible and I represents the number of infected persons.

Let R represent the number of people who have recovered from the disease. Determine, after one day, two days and three days, the values of S, I, and R. **Write a set of recursive equations describing the values of S, I, and R after n days**. We will use these later to try and predict the spread of the disease.

Simplifying Assumptions: We will assume based on the time until recovery that the rate of recovery is proportional to the number of people infected and that 1/14 of those infected will recover the next day.

Notation:

1. I_n is the number of people infected after n days
2. R_n is the number of people who have recovered after n days
3. S_n is the number of people who are susceptible after n days

We know from the info above that the initial values are

$$I_0 = 2100 \quad R_0 = 2500 \quad S_0 = 50,000 - I_0 - R_0 = 45400$$

The rate of recovery is $I/14$ and the rate of infection is $10^{-5}SI$. So we have

$$I_1 = I_0 + 10^{-5}S_0 I_0 - I_0/14$$
$$R_1 = R_0 + I_0/14$$
$$S_1 = 50,000 - I_1 - R_1$$

This is our discrete model, and so after one day:

$$I_1 = 2100 + 45400(2100)/10^5 - 2100/14 = 2100 + 953.4 - 150 = 2903$$
$$R_1 = 2500 + 150 = 2650$$
$$S_1 = 50,000 - 2903 - 2650 = 44447$$

To move to the continuous model, we must look at things interms of smaller and smaller time steps and rates of change. First, we write things in terms of change per day:

$$I_1 - I_0 = 10^{-5}S_0 I_0 - I_0/14$$
$$R_1 - R_0 = I_0/14$$
$$S_1 - S_0 = I_0 - I_1 + R_0 - R_1$$

Now we take limits (Remember DEFINITION OF DERIVATIVE):

$$I'(t) = 10^{-5}S(t)I(t) - 1/14I(t)$$
$$R'(t) = 1/14I(t)$$
$$S'(t) = -I'(t) - R'(t) = -10^{-5}S(t)I(t)$$

Predator-Prey Model and continuous modeling Here we will consider ideas that form the foundations for modern population ecology, and are due to Italian physicist-turned ecologist Vito Volterra and the American statistician Alfred Lotka (who had a lifelong interest in the biological sciences) during the mid 1920's. Your task is to write two equations built upon the assumptions given below (R = rabbit population, F = fox population).
(a) In the absence of foxes, the rabbit population grows at a constant per capita rate.
(b) The population of rabbits declines at a rate proportional to the product RF . (This assumption combines the death rate and the "those eaten" rate into one simplifying assumption.) Why might this be reasonable?
(c) In the absence of rabbits, the foxes die off at a rate proportional to the number of foxes present.
(d) The fox population grows at a rate proportional to the product of RF. Why might this be reasonable?

Questions to answer for August 8th:

1) What would be the discrete model for the predator-prey?
2) What would be the continuous model for S-I-R?
3) How are discrete and continuous models related?