MATH 34B Practice Midterm 1 Solutions

1)

1. True. The total area is the same as twice the area under $y = \sin x$ between $x = 0$ and $x = \pi$, which is the same as the signed area. Thus the area is

$$\begin{aligned} 2\int_0^\pi \sin x dx &= 2(-\cos x)|_0^\pi \\ &= 2(-\cos\pi - \cos 0) \\ &= 4 \end{aligned}$$

2. False. Put in $a = b = 1$ and see what happens.

3. True. This is the definition of frequency.

4. True. This is the Fundamental Theorem of Calculus for Definite Integrals

5. False. The Riemann sum given approximates $\int_0^1 x^2 dx$, but it does so by taking the right-hand points. Since $y = x^2$ is an increasing function, this means the Riemann sum will overcompensate, and so in fact the sum is greater than 1/3.

6. True. The function lies between 1 and 4 on the given domain, so the average must be between 1 and 4.

7. True. If i is the rate I run, m the rate my mom runs, and s the rate my sister runs, we have $i = 2m$ and $s = 3i$ from the problem. By substitution, $s = 6m$. If x is how long it takes my sister to run around the block and y how long it takes my mother to run around the block, then $x+10 = y$. Since rate times time is distances,

$$\begin{aligned} sx = my &\Rightarrow 6mx = m(10 + x) \\ &\Rightarrow 6mx = 10m + mx \\ &\Rightarrow 5mx = 10m \\ &\Rightarrow x = 2 \end{aligned}$$

Since my sister runs three times as fast as I do, that means it must take me three times as long to run around the same block, so it takes me 6 minutes.

8. True. Use Gauss' method of grouping the first and last, second and second-to-last, etc. terms together to see this.

9. True. Computing the integral on the right shows $\int_2^5 x^4 dx$ is really big, but drawing the graph of $y = \sqrt{1 - x^2}$ shows that the graph looks like the upper half of a circle (it indeed is), so the area under it must be small.

10. True. Use the chain rule and you will get the result.

2)

a) By the product rule,

$$\begin{aligned} &\frac{d}{ds}((s^2 - 2s + 3\sin 2s)(5t - ts - 7e^{s/t}) = \\ &\quad (2s - 2 + 6\cos 2s)(5t - ts - 7e^{s/t}) + (s^2 - 2s + 3\sin 2s)(-t - \frac{7}{t}e^{s/t}) \end{aligned}$$

b) We have to differentiate twice, so

$$\begin{aligned}\frac{d^2}{dt^2}\left(5\cos(\frac{71t}{\pi}+\frac{3}{\pi})\right) &= \frac{d}{dt}\left(-\frac{71}{\pi}\cdot 5\sin(\frac{71t}{\pi}+\frac{3}{\pi})\right) \\ &= -(\frac{71}{\pi})^2\cdot 5\cos(\frac{71t}{\pi}+\frac{3}{\pi})\end{aligned}$$

c) From the chain rule,

$$\begin{aligned}\frac{d}{dx}(\cos(2\pi fx+\phi))^4 &= 4(cos(2\pi fx+\phi))^3(-2\pi f\sin(2\pi fx+\phi)) \\ &= -8\pi f(\cos(2\pi fx+\phi))^3\sin(2\pi fx+\phi)\end{aligned}$$

3)

a) This integral is 0, since $\int_0^{2\pi}\cos 2tdt = 0$ and 200π is a multiple of 2π.
b)

$$\begin{aligned}\int_1^{y+\alpha}\left(3x-\frac{1}{x}\right)dx &= \left(\frac{3}{2}x^2-\ln x\right)\Big|_1^{y+\alpha} \\ &= \frac{3}{2}(y+\alpha)^2-\ln(y+\alpha)-\left(\frac{3}{2}-\ln 1\right) \\ &= \frac{3}{2}(y+\alpha)^2-\ln(y+\alpha)-\frac{3}{2}\end{aligned}$$

c)

$$\int(\sin 5x-7x^4+e^{x/3})dx = -\frac{\cos 5x}{5}-\frac{7}{5}x^5+3e^{x/3}+C$$

4) Let a be the number of fish an adult penguin eats per day and b be the number of fish a baby penguin eats per day. Then we know:

$$\begin{aligned}\text{Last year} &: 10a+2b=104 \\ \text{Few years ago} &: 5a+5b=60\end{aligned}$$

If we multiply the second equation by 2 and subtract from it the first equation, we get $8b = 16$, so $b = 2$. If we put that back into the second equation, we get $5a + 10 = 60$, or $a = 10$. Since there are now 12 adults and 3 babies, the total amount of fish eaten is

$$x = 12a+3b = 12\cdot 10+3\cdot 2 = 120+6 = 126$$