Rules: You have the full 70 minutes for the test. You are allowed **one** 3x5 notecard, and can use both sides. You may **not** use a calculator.

Test: The test will be either 6 or 7 problems of variable difficulty. It will cover all material we discuss this week, though the bulk of Wednesday will be review.

Topics: The following are the topics we have or will have covered by Wednesday that I think are most important.

(1) Trig. Know how to evaluate the basic trig functions at integer multiples of $\pi/6$ and $\pi/4$.

(2) Antiderivatives. Know the basic antiderivatives. A few that you might have forgotten but ought to know:

$$\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C.$$

(3) Riemann Sums. Know how to set up a Riemann sum to evaluate a definite integral. For example, you might be asked to use Riemann sums to compute the area under the graph $y = 2x + 1$ for $0 \leq x \leq 2$.

(4) Fundamental Theorem of Calculus. Know both forms of this. That is, know that if f is a continuous function on the interval $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

for *any* antiderivative $F(x)$, and conversely, know that if we define the function

$$g(x) = \int_a^x f(u)du$$

for some constant a, then $g'(x) = f(x)$. Thus, you need to know how to evaluate integrals, and how to find derivatives of functions such as

$$h(x) = \int_{-x}^{x^2} \sin(u)du.$$

Hint: Use the chain rule. Also, know that the fundamental theorem of calculus gives the net change theorem:

$$TotalChange = \int_a^b (RateofChange)dx.$$

Be able to use this to figure out how far a car has traveled from $t = 0$ to $t = \pi$ if its velocity is given by the function $v(t) = t\cos(t/2)$.

(5) Substitution. Know how to use substitution, which says

$$\int f'(u(x))u'(x)dx = \int f(u)du.$$

For example, know how to evaluate

$$\int xe^{x^2}\,dx.$$

Also know how to use substitution to evaluate definite integrals. For instance, be able to find

$$\int_1^4 \frac{\sin\ln x}{x}dx.$$

(6) Integration By Parts. Know how to use integration by parts, which says

$$\int u\,dv = uv - \int v\,du.$$

For example, know how to evaluate

$$\int xe^x dx, \qquad \int \ln x\,dx \qquad \int e^x \sin x\,dx.$$

(7) Average Values. Know how to compute the average value of a function. That is, the average of $f\colon [a,b] \to \mathbb{R}$ is

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx.$$

For example, find the average of xe^{-x} between -1 and 1.

(8) Areas Between Curves. Know how to find the areas between curves. That is, if $f(x) \geq g(x)$ for $x \in [a,b]$, the area between $f(x)$ and $g(x)$ between $x = a$ and $x = b$ is

$$\int_a^b (f(x) - g(x))dx.$$

For example, be able to find the area between the curves $y = x^2$ and $x = y^2$.

(9) Finding volumes. Be able to find the volume of simple shapes like pyramids, cones, and frustrums using calculus — this is material we will cover on Tuesday.

There are 90-some problems on WebWork in the optional homework set that have all of these topics represented.