Review Problems for Math 34A Spring 2009, Midterm I

The test will be 4 problems, with 2 word problems. A 3x5 notecard is allowed, but no calculators.

1) If I think there are 3000 M & M's in a jar, but there are really 3300, what is the absolute error? What is the percentage error?
The absolute error is $|3300 - 3000| = 300$. The percentage error is

$$\frac{300}{3300} \cdot 100\% = 9.09\%.$$

2) In 2000, the national gross domestic product (GDP) of Country X was \$72 billion, and in 2005 it was \$94 billion. What was the GDP in 2002?
Using linear interpolation with x equal to the year and y equal to the GDP in billions, we can calculate using the point-point equation of a line that

$$\begin{aligned} y &= \frac{94-72}{2005-2000}(x-2000)+72 \\ &= \frac{22}{5}(x-2000)+72. \end{aligned}$$

Plugging in $x = 2002$ to estimate in the year 2000, we find

$$y = \frac{22}{5}(2002-2000)+72 = \frac{44}{5}+72 = 80.8.$$

That is, we can estimate that the GDP is about \$80.8 billion.

3) Compute the following limits:

(a) $\lim_{x\to 1} x^2 - 2$

(b) $\lim_{x\to -2} \frac{x^2+x-2}{x^2+6x+8}$

(c) $\lim_{x\to\infty} \frac{7x-102}{3x+22}$.

(a) Plugging in 1 directly, we see that the limit is -1.
(b) We first simplify

$$\frac{x^2+x-2}{x^2+6x+8} = \frac{(x+2)(x-1)}{(x+2)(x+4)} = \frac{x-1}{x+4}.$$

Plugging in -2 directly, we then see that the limit is $-3/2$.
(c) When x is really big, we notice that

$$\frac{7x-102}{3x+22} \approx \frac{7x}{3x} = \frac{7}{3}.$$

This means that the limit is $7/3$.

4) A grain silo is in the shape of a cylinder with a hemisphere (half a sphere) on top. If the total volume is 1000 m^3, express the surface area in terms of the radius of the silo.

Let r be the radius of the silo and h be the height of the silo. Then we know that the surface area A is given by

$$A = 2\pi rh + 2\pi r^2,$$

where the first part is the area of the curved part of the silo and the second part is the area of the hemisphere (that is, half the surface area of a sphere). We want to eliminate h, so we use the volume equation

$$1000 = \pi r^2 h + \frac{2}{3}\pi r^3.$$

Solving for h in terms of r yields

$$\begin{aligned} \pi r^2 h &= 1000 - \frac{2}{3}\pi r^3 \\ h &= \frac{3000 - 2\pi r^3}{3r^2}. \end{aligned}$$

Substituting back into the equation for the surface area gives

$$A = \frac{6000 - 4\pi r^3}{3r} + 2\pi r^2,$$

which is the desired answer.

5) Two bicyclists leave campus at noon. One bikes north at 20 mph, and the other bikes west at 5 mph. Assuming they don't stop, when are they 15 miles apart?
Let t be the time (in hours) after noon, let d_n be the distance from campus of the biker who heads north, let d_w be the distance from campus of the biker who heads west, and let d be the distance between the two bikers. Then using "distance equals rate times time," we see that

$$d_n = 20t \qquad d_w = 5t.$$

Using the Pythagorean Theorem (draw a picture!), we then find that the distance between the bikers is

$$\begin{aligned} d^2 &= d_n^2 + d_w^2 \\ &= 400t^2 + 25t^2 \\ &= 425t^2. \end{aligned}$$

Taking the square root of both sides yields

$$d = \sqrt{425}t.$$

6) 10 adults and 5 children can watch a movie for \$75, but 8 adults and 10 children can watch the movie for \$78. How much would it cost for 6 adults and 10 children to watch the movie?
Let A be the cost of an adult ticket and C be the cost of a child ticket. We then know that

$$\begin{cases} 10A + 5B &= 75 \\ 8A + 10B &= 78. \end{cases}$$

If we multiply the top equation by 2 and then subtract the two equations, we see that

$$12A = 72,$$

and so $A = 6$. Plugging back into the first equation, we see that $60 + 5B = 75$, or $B = 3$. That is, it costs \$6 per adult and \$3 per child to watch the movie. So if there are \$6 adults and \$10 children, it will cost \$66.

7) I have two bottles of sugar water. The first bottle is 10% sugar and 90% water. The second is 20% sugar and 80% water. How much of each bottle should I use to get 1 liter of liquid which is 13% sugar?
Let A be the amount of the first bottle used and let B be the amount of the second bottle used. Then writing equations for the total amount of liquid and the total amount of water, we see that

$$\begin{cases} A + B & = 1 \\ 10A + 20B & = 13. \end{cases}$$

Multiplying the top equation by 20 and subtracting, we find that $10A = 7$, or $A = 0.7$. Plugging back into the first equation, we see that $B = 0.3$. That is, we should use 0.7 liters from the first bottle and 0.3 liters from the second bottle.

8) A manufacturer sells lamps at \$6 each and sells 3000 each month. For each \$1 that the price is increased, 1000 fewer lamps are sold each month. It costs \$4 to make one lamp. Express the total profit in terms of the price of a lamp.
Let P be the total profit, let x be the price of a lamp, and let n be the number of lamps sold. Writing the equation of the line n in terms of x, we know that the slope is given by -1000 (the change in x is 1 and the change in n is -1000). We also know that the point $(6, 3000)$ is on the line. Thus, using the point-slope equation of a line, we have

$$n = -1000(x - 6) + 3000 = -1000x + 9000.$$

On the other hand, profit is equal to the profit per lamp times the number of lamps sold. Thus the profit is

$$\begin{aligned} P &= (x - 4)n \\ &= (x - 4)(-1000x + 9000) \\ &= -1000x^2 + 4000x + 9000x - 36000 \\ &= -1000x^2 + 13000x - 36000. \end{aligned}$$