Review Problems for Math 34A Spring 2009, Midterm II

The test will be 4 problems, with 2 word problems. A 3x5 notecard is allowed, but no calculators. The test will be on the material in Chapter 7.

1) Use the logarithm rules to write the following with as few logarithms as possible:

(a) $\log(x) + 3\log(y) - 2 = \log\left(\frac{xy^3}{100}\right)$.

(b) $\frac{\log(3) - \log(7)}{\log(4) + \log(3)} = \frac{\log(3/7)}{\log(12)}$.

(c) $\frac{\log(x) - \log(xy)}{\log(y^3) + \log(y)} = \frac{-\log(y)}{4\log(y)} = \frac{1}{4}$.

2) Solve for x in the following equations. Your answer should involve only the logarithm of integers and variables (so $\log(2) - \log(3)$ is acceptable, but $\log(2/3)$ is not).

(a) $3^x = 24$.

Solution: Taking the log of both sides and solving, we see that

$$x = \frac{\log 24}{\log 3}.$$

(b) $7 \cdot 8^x = 3^{x-1}$.

Solution: Taking the log of both sides and using the log rules yields

$$\log 7 + x\log 8 = x\log 3 - \log 3.$$

Solving for x then yields

$$x = \frac{\log 7 + \log 3}{\log 3 - \log 8}.$$

(c) $4 \cdot 5^x = 2y$.

Solution: Taking the log of both sides yields

$$\log 4 + x\log 5 = \log 2 + \log y.$$

Solving for x yields

$$x = \frac{\log 2 + \log y - \log 4}{\log 5} = \frac{\log y - \log 2}{\log 5}.$$

3) Change the following exponentials to have base e (that is, write in the form e^y), using the natural logarithm:

(a) 3^x

Solution: We want to solve for y in the equation $e^y = 3^x$. Taking the natural log of both sides yields

$$y = x\ln 3.$$

(b) 2^{x-1}

Solution: We want to solve for y in the equation $e^y = 2^{x-1}$. Taking the natural log of both sides yields

$$y = (x-1)\ln 2.$$

4) Initially I have 10g of a certain radioactive isotope. Three hours later I have 3g left. What is the half-life, in hours?

Solution: Let A denote the amount of the radioactive isotope and let t be the amount of time that has passed, in hours. The half-life formula is

$$A(t) = 10\left(\frac{1}{2}\right)^{t/k},$$

where k is the half-life, in hours. We know that $A(3) = 3$, so

$$\begin{aligned} 3 &= 10\left(\frac{1}{2}\right)^{3/k} \\ \frac{3}{10} &= \left(\frac{1}{2}\right)^{3/k} \\ \log(3/10) &= \frac{3\log(1/2)}{k} \\ k &= \frac{3\log(1/2)}{\log(3/10)}. \end{aligned}$$

That is, the half-life is $\frac{3\log(1/2)}{\log(3/10)}$ hours.

5) John's credit card has an APR of 10%, and interest is compounded monthly. He charges a \$1000 computer to his card. Being lazy, John neglects to pay off the card for an entire year. Assuming there are no additional fees, what does he owe after a year has passed?

Solution: Since the initial amount is \$1000 and the APR is 10% and interest is compounded monthly (twelve times a year), we know that the amount on his credit card is

$$A(t) = 1000\left(1 + \frac{10}{12}\cdot\frac{1}{100}\right)^{12t} = 1000\left(\frac{121}{120}\right)^{12t},$$

where t is the number of years that have passed. Thus, after one year, John owes

$$A(1) = 1000 \cdot \frac{121}{120} \text{ dollars.}$$

6) A mold colony develops on a piece of bread. If the colony doubles in size every 4 days, how many days until it is ten times its original size?

Solution: Since the doubling time is 4 days, we know that the size of the colony is given by

$$A(t) = A_0 \cdot 2^{t/4},$$

where A_0 is the initial size and t is the number of days that have passed. We want to know when the amount is ten times the original amount, or $10A_0$. So we solve

$$\begin{aligned}
10A_0 &= A_0 \cdot 2^{t/4} \\
10 &= 2^{t/4} \\
\log 10 &= \frac{t \log 2}{4} \\
t &= \frac{4 \log 10}{\log 2}.
\end{aligned}$$

7) Two bacteria cultures are growing on the same petri dish. Initially the first culture has a mass of 1mg and the second has a mass of 2mg. Twenty days later, both cultures have the same mass. If the mass of the first culture doubles every 5 days, what is the doubling time, in days, of the second culture?

Solution: Let $A(t)$ be the mass of the first culture and $B(t)$ be the mass of the second culture. We don't know the doubling time of the second culture, so let k denote the unknown doubling time. The doubling time formulas give us

$$\begin{aligned}
A(t) &= 2^{t/5} \\
B(t) &= 2 \cdot 2^{t/k}.
\end{aligned}$$

Additionally, we know that $A(20) = B(20)$ because the populations are the same after twenty days. Using the formulas for A and B, we see that

$$\begin{aligned}
2^{20/5} &= 2 \cdot 2^{20/k} \\
2^4 &= 2 \cdot 2^{20/k} \\
2^3 &= 2^{20/k} \\
3 &= \frac{20}{k} \\
k &= \frac{20}{3}.
\end{aligned}$$

8) I have a pool of money in my bank account. Ten years ago I had \$1000 in the account. Now I have \$1500 in the account. Assuming interest is compounded annually, what is the interest rate on my bank account?

Solution: Since the initial amount is \$1000 and interest is compounded annually, we know that

$$A(t) = 1000(1 + \frac{I}{100})^t,$$

where I is the interest rate. We know also that $A(10) = 1500$, so

$$\begin{aligned}
1500 &= 1000\left(1 + \frac{I}{100}\right)^{10} \\
\frac{3}{2} &= \left(1 + \frac{I}{100}\right)^{10} \\
\left(\frac{3}{2}\right)^{1/10} &= 1 + \frac{I}{100} \\
I &= 100\left(\left(\frac{3}{2}\right)^{1/10} - 1\right).
\end{aligned}$$