MAT 175 HOMEWORK #5

DUE APRIL 5 (WEDNESDAY)

Note: *Please* **staple** *if necessary. Numbering of problems is as in the textbook.*

(12.2.**18**) Let $f(x,y) = (x^3 + y^2)^5$. Verify that

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

(12.2.**34**) Show that $f(x,y) = \ln(4x^2 + 4y^2)$ satisfies **Laplace's Equation**:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

(12.3.**4**) Find the indicated limit or state that the limit does not exist.

$$\lim_{(x,y)\to(1,2)} \frac{x^3 - 3x^2y + 3xy^2 - y^3}{y - 2x^2}$$

(12.3.**10**) Find the indicated limit or state that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}$$

(12.3.**36**) Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^2}$$

does not exist by considering one path to the origin along the x-axis and another path along the line $y = x$.

(12.4.**12**) Find the gradient vector of the function

$$f(x,y) = x^3y + 3xy^2$$

at the point $\mathbf{p} = (2,-2)$. Then find the equation of the tangent plane at $\mathbf{p}$.

(12.4.**14**) Find the gradient vector of the function

$$f(x, y) = \frac{x^2}{y}$$

at the point $\mathbf{p} = (2, -1)$. Then find the equation of the tangent plane at $\mathbf{p}$.

(12.5.**14**) In what direction $\mathbf{u}$ does $f(x, y) = \sin(3x - y)$ decrease most rapidly at $\mathbf{p} = (\pi/6, \pi/4)$?

(12.5.**16**) Sketch the level curve of $f(x, y) = x^2+4y^2$ that goes through $\mathbf{p} = (2, 1)$. Calculate the gradient $\nabla f(\mathbf{p})$ and draw this vector, placing its initial point at $\mathbf{p}$. What should be true about $\nabla f(\mathbf{p})$?

(12.5.**18**) Find the directional derivative of $f(x, y) = e^{-x} \cos y$ at $(0, \pi/3)$ in the direction toward the origin.