

### Problem Solving Strategies.

Here are some things which will help you solve unfamiliar problems. See also sections 17.4 and 17.11.

- (1) Read the question several times to make sure you really understand what is being asked. Make sure you take note of **all** the information you are given.
- (2) Write down in **plain English** what you are trying to find; what your **goal** is. Sometimes students, in their haste, try to find the wrong thing.
- (3) Give symbolic names (like  $w$  or  $x$ ) to any quantities (unknowns) in the problem. Write down very clearly what each letter represents, even if it seems obvious to you that  $w$  is the **width**. In such a case you should draw a diagram, showing what the width is, so you do not confuse it with the length.
- (4) Draw **neat** diagrams clearly showing as much information as possible.
- (5) Write in **complete sentences**, not short-hand or key phrases, thoughts about the problem, in a “**thinks box**”.
- (6) Avoid pronouns: **it, this, that** try to be specific and clear.
- (7) Try explaining to a friend in **very simple and precise** terms what you understand and where you are stuck.
- (8) Never say stuff like “this is the  $y$  for  $x$ ” or “you plug this in here and move that there.”
- (9) Try to imagine yourself **in the problem**. For example, if a problem involves two cars, imagine yourself in one of the cars.
- (10) If you can not solve the given problem, try to make a simpler related problem that you can solve. If you can solve the simpler problem, perhaps you will get ideas for the original problem. I often do this when a problem has lots of unknowns in it. For example if a car is moving with speed  $v$  I might first work the problem with speed 10 and see if I can do that. If I can, then probably changing the 10 to  $v$  in a few places will solve the original.
- (11) Try to use your **common sense** as much as possible. Everyone is a lot smarter than they think, if they use all their mental abilities. Common sense is far more useful than a formula.
- (12) Make a **plan**. Examples of plans are given for the next 8 problems.

We will illustrate problem solving by giving some examples together with an **outline** or **plan** of the solution method. This outline will be at a higher level than “set  $A$  equals  $L$  times  $W$  and now plug in  $W = 3$  and  $A = 6$  and solve for  $L$ .” These instructions do indeed tell us how to get to the answer  $L = 2$ . But, they do not help us understand the bigger picture. The following examples are intended to be **models** of how to think about problems. This is **high level** thinking. It is hard. You will find it easier at first to do this with the help of a friend or two.

**Problem 3.1.1** *There are three consecutive numbers. The sum of these numbers is 300. Find the numbers.*

The first step is to give a name to the first number as it is the unknown in this problem. Once we have named the first number, the other two numbers can be expressed in terms of this single unknown. The condition that these three numbers add up to 300 can now be expressed as an equation which involves just this unknown. Solve the equation. Use the solution to write down the three numbers. Check the answer is correct.

**Problem 3.1.2** *Liquid A has 10 grams of element X per liter. Liquid B has 15 milligrams of element Y per  $\text{cm}^3$ . It is desired to make 100 grams of molecule Z. To make 18 grams of Z requires 2 grams of X and 16 grams of Y. How much of the two liquids must be used?*

Convert everything into the same units. Figure out how many grams of X and Y are needed to make 100 grams of Z. Figure out how much each of the two liquids is needed to get these amounts of X and Y. Check your answer.

**Problem 3.1.3** *A rectangle is twice as wide as it is long. If the area is 450 square meters, what are the dimensions of the rectangle?*

The length and width are the unknowns. We obtain one equation which says the width is twice the length. We get another equation using that the area is the product of length and width. This gives two equations involving two unknowns. Solve them. Check the answer is correct.

**Problem 3.1.4** *Express the area of a square in terms of the perimeter.*