Version 3/7/07 - still subject to mild revisions

Coverage for our Final 12-3pm, Thursday, March 22, 2007

Definitions:

From Midterm:

Outer measure on X, the measure generated by an outer measure, the measurable sets with respect to an outer measure, distance between sets, metric outer measures, Hausdorff measures, Hausdorff dimension, σ-finite, $L^p(\mu)$ for $1 \leq p \leq \infty$, convergence in measure, Cauchy in measure, the spaces l_n^p, l^p, l_d^p, and c_0 as used in the discussion of Fourier series, normed linear space, inner-product space, Banach space, Hilbert space, norm of an operator, dual space, Fourier series, trigonometric polynomial, Dirichlet kernels.

Since Midterm:

The convolution $f * g$ of f and g, sublinear function on a vector space, uniform, strong and weak convergence of a sequence of operators, signed and complex measures, total variation measure of a complex measure, the positive and negative variations of a signed measure, "λ is absolutely continuous with respect to μ" ($\lambda << \mu$), "λ is concentrated on A," "λ_1 and λ_2 are mutually singular" ($\lambda_1 \perp \lambda_2$), $Q_r\mu$, $D\mu$, $M\mu$, Mf, lower semicontinuous, Lebesgue points, shrinks nicely, absolute continuity of $f : [0, 1] \to \mathbb{C}$, the total variation of $f : [0, 1] \to \mathbb{C}$ on $[0, x]$.

Statements and Proofs: A fair number of the proofs are too long to ask for on an exam, but they have "parts" which can be called for, thereby allowing allow you to demonstrate your mastery of the full proofs.

From Midterm:

Statement of Theorem 2.3, statements and proofs of Propositions 2.6, 2.7, Theorem 2.8, Corollary 2.9 statement of Fatou's Lemma, statement of Fatou's Lemma, statement and proof via Fatou's Lemma of Lebesgue Dominated Convergence Theorem as formulated in the notes, statement and proof of Hölder's inequality and Minkowski's inequality, proof that $L^p(\mu)$ is complete, including statement and proof of Lemma 3.3, statements and proofs of Theorem 3.5 and Theorem 4.4, statement and proof of the Banach Steinhaus Theorem/Uniform Boundedness Principle, statement only of the Open Mapping Theorem, statements and proofs of Lemma 5.1 (Riemann-Lebesgue Lemma) and Theorem 5.3 (you may assume $\int |D_N| \to \infty$), statement and proof of Theorem 5.4, statement and proof of the Cauchy-Schwarz inequality and all the results in the appendix on inner-product and Hilbert spaces up to, but not including, the Riesz Representation Theorem.

Since Midterm:

Statement and proof of the Riesz Representation Theorem, Corollary 11.8 (current numbering; label "rrep"), statement and proof of the convolution theorem Theorem 6.1 (you may ignore the measurability issue), statement only Theorem 6.2 (but note that Exercise 39 is listed under the exercise section below), statement and proof of the Mazur-Orlicz Lemma Theorem 7.1, statement and proof of the Hahn-Banach Extension Theorem 7.5, statement and proof of Theorem 8.3, statement only of Theorem 8.6, statement only of the Lebesgue-Radon-Nikodym Theorem 8.11, statement only of Lemma 8.17, statement of Theorem 8.23 and the proof in the case $\mu(X) < \infty$, statements and proofs of Lemma 9.8 and Theorem 9.7, statement and proof of Theorem 9.4, statements and proofs of the results in Section 9.2.1, statements only of Theorems 10.3, 10.5, 10.7.

Examples:

From Midterm:

Example of a sequence of functions converging to zero in $L^p(0,1)$, $1 \leq p < \infty$, which does not converge a.e. to zero,

Since Midterm:

Examples of sequences of operators which converge uniformly, strongly and not uniformly, weakly and not strongly, example of a nonzero measure ν for which $\nu \perp \mathcal{L}^n$, where $\mathcal{L}^n$ is Lebesgue measure, example of an everywhere differentiable function $f : \mathbb{R} \to \mathbb{R}$ whose variation on $[0, x]$ is infinite for $0 < x \leq 1$, (you should be able to prove that $f(t) = t^2 \sin(\pi/(2t^2))$, $f(0) = 0$, works.)

Exercises:

From Midterm (there have been deletions):

3, 10, 11, 12, 13, 18, 19 ((i), (ii), (v)),

Since Midterm:

Redo version of Problem 24, 31, 35, 39

Other Stuff: We may pose other problems which should seem reasonable if you know the above stuff.