1. PROBLEM ASSIGNMENTS

When writing up the hand-in homeworks (TI problems below), do at least the following:

- Include a version of the problem statement so that a reader can tell what you are doing without looking elsewhere.
- Label all the variables you use, if this is appropriate to the problem. More generally, explain any notation that you use that is not completely standard.
- Explain what you are doing in words, don't just suddenly write equations, etc. We ask that you to write in complete sentences rather than leave out verbs, subjects, etc.
- Turn in a good looking document. It represents you, and if you write diagonally on the page, put stuff in random places, scratch things out, write unreadably messy or small, it doesn't represent you well and causes anyone who attempts to read your work to snort, mutter unpleasant things, and delete points.

Remember that the turned in homework is to be on one side only of 8 by 11.5 inch paper which is not ripped from a spiral binder, your name and section time is to be on every page, and the pages are to be stapled at the upper-left hand corner.

Only a few problems can actually be graded, and then presentation will count, per the above. A point will be given for every problem you have seriously attempted, whether or not it is actually graded.

Remember that "Don't Turn In" problems, or analogues of them, may occur on quizzes and exams. I will assign most straight computational problems under "Don't Turn In;" answers are in the book. Do as many of these as you need to do in order to be able to tell by a look at the others than you can do those too - and then do a few more to prove that you are right.

<u>TI Problems Due April 9</u>

TI Problem 1. Give an example of a subset of \mathbb{R}^2 which is neither open nor closed and contains both interior points and at least one boundary point. Give an explicit example of an interior point of your set and exhibit a $\delta > 0$ for which the δ neighborhood of the point is contained in the set. Give an explicit example of a boundary point of the set which lies in the set, and verify that it is indeed a boundary point.

TI Problem 2. Determine whether or not each of the following limits exist. If the limit exists, give the value of the limit and justify your answer by explicitly quoting applicable

results. If the limit does not exist, justify that assertion.

(a)
$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{1 + x + y},$$

(b)
$$\lim_{(x,y)\to(1,0)} \frac{(x-1)^2 - y^2}{x + y - 1},$$

(c)
$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2},$$

(d)
$$\lim_{(x,y)\to(0,0)} \frac{1}{x^2 + y^2}.$$

TI Problem 3. Suppose that $D \subset \mathbb{R}^2$ is a domain and $f : D \to \mathbb{R}$ is continuous. If $(x_1, y_1) \in D$ and $f(x_1, y_1) = 1$, show that there is a δ -neighborhood of (x_1, y_1) which is contained in D and such that f(x, y) > 1/2 for all (x, y) in this neighborhood.

Don't Turn in Problems Due April 9

Text Pg 82: 2 a), b), 6, 10 a), b) and Pg 88: 1.

TI Problems Due April 16

TI Problem 4. Given that

$$r = \sqrt{x^2 + y^2}, \ x = r\cos(\theta)$$

evaluate all of

$$\left(\frac{\partial r}{\partial x}\right)_{y}, \ \left(\frac{\partial r}{\partial x}\right)_{\theta}, \ \left(\frac{\partial r}{\partial \theta}\right)_{x}, \ \left(\frac{\partial r}{\partial \theta}\right)_{y}, \ \left(\frac{\partial \theta}{\partial r}\right)_{y}, \ \left(\frac{\partial \theta}{\partial x}\right)_{r}.$$

You may assume that $0 < y, 0 < \theta < \frac{\pi}{2}$.

TI Problem 5. Let u = f(x, y), v = g(x, y) satisfy f(0, 0) = g(0, 0) = 0 and

(1.1)
(i)
$$-x + u + 2v + e^{(u+v)} = 1,$$

(ii) $y + uv + \sin(3u - v) = 0,$

in some neighborhood of (0,0). Here (1.1) means

(i)
$$-x + f(x, y) + 2g(x, y) + e^{(f(x,y) + g(x,y))} = 1,$$

(ii) $y + f(x, y)g(x, y) + \sin(3f(x, y) - g(x, y)) = 0,$

for (x, y) in the neighborhood.

Assume that u and v have continuous partial derivatives in the neighborhood.

- (a) Differentiate each equation in (1.1) with respect to x and solve the resulting system of linear equations in $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}$ to express $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}$ in terms of x, y, u, v. (b) Plug (x, y) = (0, 0) in the result of (a) to compute $\frac{\partial u}{\partial x}(0, 0), \frac{\partial v}{\partial x}(0, 0)$.

- (c) Repeat the procedure in order to compute $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$, except this time plug in (x, y) =(0,0) before solving for $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial y}$ and then solve for $\frac{\partial u}{\partial y}(0,0)$, $\frac{\partial v}{\partial y}(0,0)$. (d) Give an approximation to u(.01, -.03), v(.02, -.01).

TI Problem 6. Problem 4 of the text, page 96.

Don't Turn in Problems Due April 16

Pg 89: 1-5 and Pg: 95, 1-3.

TI Problems Due April 23

TI Problem 7. Problem 8, page 100 of the text.

TI Problem 8. Suppose $f : \mathbb{R}^2 \to \mathbb{R}^3$, $g : \mathbb{R}^4 \to \mathbb{R}^2$, g(0, 0, 0, 0) = (1, 1) and

$$Dg(0,0,0,0) = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 \end{bmatrix}, \ Df(1,1) = \begin{bmatrix} \pi & 6 \\ -3 & 5 \\ 7 & 11 \end{bmatrix}.$$

Suppose h(x, y, z, t) = f(g(x, y, z, t)). Compute $\frac{\partial h}{\partial z}(0, 0, 0, 0)$.

TI Problem 9. The transformation which take spherical coordinates into Cartesian coordinates is

$$(x, y, z) = f(\rho, \phi, \theta) = (\rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi)).$$

Compute the Jacobian matrix $Df(\rho, \phi, \theta)$ and its determinant det(Df). Let

$$(\Delta x, \Delta y, \Delta z) = f\left(1 + \Delta \rho, \frac{\pi}{2} + \Delta \phi, \Delta \theta\right) - f\left(1, \frac{\pi}{2}, 0\right)$$

If $\Delta \rho = -.001$, use linear approximation to give values of $\Delta \phi, \Delta \theta$ for which $\Delta y \approx$ $\Delta x, \Delta z \approx 2\Delta y$. Here " \approx " means "just about equal." Then compute $\Delta x, \Delta y, \Delta z$ to 6 decimal places for this choice of $\Delta \rho, \Delta \phi, \Delta \theta$ (calculator or computer or tables - say what you used).

Don't Turn in Problems Due April 23

Pg 100: 1-6, Pg 104: 1, 2, 4, Pg 116: 1-4, 6.

TI Problems Due April 30

TI Problem 10. Suppose $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3), z = (z_1, z_2, z_3) \in \mathbb{R}^3$ and

(1.2)
$$D = \det \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \neq 0.$$

When (1.2) holds, for any $w = (w_1, w_2, w_3) \in \mathbb{R}^3$, there are unique numbers $c_1, c_2, c_3 \in \mathbb{R}$ such that $w = c_1 x + c_2 y + c_3 z$, which is the same as

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = c_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + c_2 \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + c_3 \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix};$$

that is, x, y, z is a basis for \mathbb{R}^3 . We will say that the basis x, y, z, written in that order, is positively oriented if D > 0 in (1.2).

- (a) Show that $\vec{i}, \vec{j}, \vec{k}$ is positively oriented.
- (b) Let x, y, z be a positively oriented basis of \mathbb{R}^3 . Explain why x, z, y is not positively oriented. List all of the reorderings of x, y, z which are positively oriented.
- (c) If $x, y \in \mathbb{R}^3$ and $x \times y \neq 0$, show that $x, y, x \times y$ is positively oriented. Hint: the relevant determinant has the value $|x \times y|^2$; make a smart choice of a row or column about which to expand the determinant in order to see this easily.

TI Problem 11. Find the equations of the planes \mathcal{P} in \mathbb{R}^3 satisfying the given conditions. In each case, give two explicit points which are in the plane and not among the data given.

- (i) \mathcal{P} contains the points (0, 1, 0), (1, 2, 0), (1, 1, 1)
- (ii) \mathcal{P} is parallel to the plane 2x 3y + 5z + 17 = 0 and contains the point (1, 1, 0).
- (iii) \mathcal{P} is parallel to both of the lines $t \mapsto t(3,4,5)$ and $t \mapsto (1,2,3) + t\vec{i} 2\vec{j}$ and contains the point (1,1,0).
- (iv) \mathcal{P} is parallel to the line $t \mapsto t(3, 4, 5)$ and contains the points (1, 1, 0), (3, 2, 1).
- (v) \mathcal{P} is perpendicular to the line $t \mapsto (1,2,3) + t\vec{i} 2\vec{j}$ and contains (1,-1,1).
- (vi) Find the point where the line given in (v) intersects the plane you found.

TI Problem 12. Let $N \in \mathbb{R}^3$, $N \neq 0$. Consider the plane \mathcal{P} whose equation is

(1.3)
$$N \cdot (x - x_0, y - y_0, z - z_0) = 0.$$

If $P \in \mathbb{R}^3$ is not in \mathcal{P} , find the one and only point $Q \in \mathcal{P}$ such that P - Q is a multiple of N, that is, P - Q is also a normal to the plane. Hints: Write $P = \alpha N + Q$ and assume that $Q \cdot N = N \cdot (x_0, y_0, z_0)$ (this puts Q in the plane). Now determine α by taking the inner-product of both sides of $P = \alpha N + Q$ with N. Show that the Q you compute this way is in fact in \mathcal{P} . Then show that

(1.4)
$$|P - Q| = \frac{|(P - (x_0, y_0, z_0)) \cdot N|}{\|N\|}.$$

If \mathcal{P} is given by Ax + By + Cz + D = 0 and $P = (x_1, y_1, z_1)$, show that (1.4) just says that

(1.5)
$$|P - Q| = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Draw a picture which shows why either (1.4) or (1.5) give the distance from P to \mathcal{P} . Calculate the distance from (1, 2, -3) to the plane $(6, 7, 8) \cdot (x, y, z) = 7$.

Don't Turn in Problems Due April 30

Pg 15: 1, 2, 6, 7, 8, 11, Pg 121: 2-4, 7.

NTI Handout Problem 1. Compute the following:

(i) $x \cdot y$ where $x = (1, 3, -5, 7), y = (\pi, e, 6, 1).$

- (ii) The cosine of the angle between x, y above.
- (ii) $(3\vec{i} 4\vec{k} + \vec{j}) \cdot (3\vec{k} 4\vec{j} + \vec{i})$ (read this carefully).
- (iv) $(3, 1, -4) \times (1, -4, 3)$. (v) $(3\vec{i} 4\vec{k} + \vec{j}) \times (3\vec{k} 4\vec{j} + \vec{i})$ (read this carefully).
- (vi) The direction of (1, 3, -2).
- (vi) The direction of (1, 3, -2, 1).
- (vii) The direction of $\vec{i} + 3\vec{j} 2\vec{k}$.

TI Problems Due May 7

TI Problem 13. Let $u(t) = (u_1(t), \ldots, u_n(t)), \alpha \le t \le \beta$, be a curve in \mathbb{R}^n . One defines

(1.6)
$$\frac{du}{dt} = \left(\frac{du_1}{dt}, \dots, \frac{du_n}{dt}\right)$$

Suppose that $v(t) = (v_1(t), \dots, v_n(t))$ is another curve and $g : \mathbb{R} \to \mathbb{R}$.

(a) Show that

$$\frac{d}{dt}(u(t)\cdot v(t))) = \left(\frac{du}{dt}\right)\cdot v + u\cdot \left(\frac{dv}{dt}\right).$$

- (b) Use part (a) to show that |u(t)| is constant if and only if $\frac{du}{dt} \perp u$.
- (c) Show that

$$\frac{d}{dt}(g(t)u(t)) = g'(t)u(t) + g(t)\frac{d}{dt}u(t)$$

(d) If n = 3, show that

$$\frac{d}{dt}(u \times v) = \frac{du}{dt} \times v + u \times \frac{dv}{dt}.$$

TI Problem 14. There are dragons on the planet Dragone. You are riding a land rover on Dragone at its maximum speed of 20 klicks/hour. In some coordinate system, by empirical measurements, it is estimated that the danger due to dragons is given by $D = e^{xy^2} + 1/(x^2 + 1)$ (x, y are measured in klicks). You are at the position (2, 1). Which direction should you head in in order to reduce D as fast as possible? At what rate is Ddecreasing if you are headed in this direction at your maximum speed? Use a calculator or computer to give a decimal number as an answer here.

TI Problem 15. Problem 2, page 135, of the text.

Don't Turn in Problems Due May 7

Pg 127: 1, 2, 10, and Pg 134: 1, parts a)-e), and Page 143: 1 - 4, 10.

NTI Handout Problem 2. Find the directional derivative of

$$f(x_1, \dots, x_n) = x_1 x_2^2 x_4^3 - x_3$$

in the direction of the outward pointing normal to the surface

$$x_1^2 + x_2^2 + x_3^2 + 8x_4^4 = 654$$

at the point (-2, 1, 1, -3) on the surface.

TI Problems Due May 14

TI Problem 16. Problem 17 of the text, Pg 187

TI Problem 17. Problem 18 of the text, Pg 187

TI Problem 18. Problem 16 of the text, Pg 187

Don't Turn in Problems Due May 14

Pg 180: 6, 7, Pg 185: 3, 5, 6.

<u>TI Problems Due May 21</u>

TI Problem 19. Let C be given in terms of polar coordinates by $r = g(\theta), \theta_1 \le \theta \le \theta_2$. Show that if f(x, y) is a function of x, y, then

$$\int_C f \, ds = \int_{\theta_1}^{\theta_2} f(r\cos(\theta), r\sin(\theta)) \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta.$$

TI Problem 20. Sketch the cardioid, which is given by

$$r = 1 + \cos(\theta), \ 0 \le \theta \le 2\pi.$$

Compute the length of the cardioid.

TI Problem 21. Let *C* be the parameterized curve given by $x = \phi(t), y = \psi(t), h \le t \le k$. Let $g : [a, b] \to [h, k]$ be continuously differentiable. Here $g : [a, b] \to [h, k]$ means that if $a \le \tau \le b$, then $h \le g(\tau) \le k$. Let \hat{C} be the parameterized curve given by $x = \phi(g(\tau)), y = \psi(g(\tau)), a \le \tau \le b$. We consider two cases:

- (i) g(a) = h, g(b) = k, (orientation preserving)
- (ii) g(a) = k, g(b) = h, (orientation reversing).
- (a) Give examples of an orientation preserving $g : [2,3] \to [0,1]$ and an orientation reversing $g : [2,3] \to [0,1]$. Hint: you can do this with linear g's.

(b) Assume that g' is either always nonnegative or always nonpositive on [a, b]. If f(x, y) is defined on the trace of C, then

$$\int_{\hat{C}} f \, ds = \int_{C} f \, ds$$

holds if g is orientation preserving and if g is orientation reversing.

(c) If P(x, y) is defined on the trace of C, then

$$\int_{\hat{C}} P \, dx = \pm \int_{C} P \, dx$$

where the plus sign is correct if g is orientation preserving and the minus sign is correct if g is orientation reversing.

Big Hint: if $G : [h, k] \to \mathbb{R}$, then, by Math 3B,

$$\int_{a}^{b} G(g(\tau))g'(\tau) \, d\tau = \int_{g(a)}^{g(b)} G(t) \, dt.$$

<u>Don't Turn in Problems Due May 21</u>

Pg 278, 1-4.

TI Problems Due May 30

TI Problem 22. A thin plate occupies the region $1 \le x \le 2, 0 \le y \le \ln(x)$, of the x, y plane. The units of x, y are inches. The density of gold in the plate is $(x - 1)\sqrt{1 + e^{2y}}$ ounces of gold per square inch. Compute the total amount of gold in the plate. Hint: Consider changing the order of integration and use substitutions and tables.

TI Problem 23. Find the volume bounded by the intersection of the cylinders

$$x^2 + y^2 = a^2$$
, $x^2 + z^2 = a^2$,

where a > 0.

Don't Turn in Problems Due May 30

Pg 234: 1-5, 6 a,b, Pg 241: 4.

<u>TI Problems Due June 6</u>

None. Work on the final preparation handout.

Don't Turn in Problems Due June 6

Pg 286: 1-3, 5, 7, Pg 300: 1-3.