MATH 3B REVIEW SECTION

DANNING LU
DANNING.LU@MATH.UCSB.EDU

1. Area between curves

Find the area between the curves $f(x)$ and $g(x)$ where:

(1) $f(x) = -x^3/2 + 2x^2$, $g(x) = -x^2 + 4x$, $-1 \leqslant x \leqslant 3$.
(2) $f(x) = \cos x$, $g(x) = \sin 2x = 2 \sin x \cos x$, $0 \leqslant x \leqslant \pi/2$.
(3) $f(x) = \arccos x$, $g(x) = \arcsin x$, $-\pi/2 \leqslant x \leqslant \pi/2$.

Answer:

(1) Zero points: $x = 0, 2, 4$.
$-1 < x < 0$: $f - g$ positive.
$0 < x < 2$: $f - g$ negative.
$2 < x < 3$: $f - g$ positive.
Final answer: 6.
(2) Zero points: $x = \pi/6, \pi/2$.
$0 < x < \pi/6$: $f - g$ positive.
$\pi/6 < x < \pi/2$: $f - g$ negative.
Final answer: 1/2.
(3) Zero points: $x = \pi/4$.
$-\pi/2 < x < \pi/4$: $f - g$ positive.
$\pi/4 < x < \pi/2$: $f - g$ negative.
Final answer: 1/2.

2. Integrate by parts

(1) Integrate $\int_4^{4\sqrt{3}} 6\arctan(8/x)dx$.
(2) Integrate $\int (3t+t^2)\sin(2t)dt$.
(3) Integrate $\int t^7 \sin(2t^4)dt$.

Answer:

(1)
$$\int_4^{4\sqrt{3}} 6\arctan(8/x)dx = 24(\ln\frac{7}{5} - \arctan 2 + \sqrt{3}\arctan\frac{2}{\sqrt{3}}).$$
(2)
$$\int (3t+t^2)\sin(2t)dt = \frac{-2t^2-6t+1}{4}\cos(2t) + \frac{2t+3}{4}\sin(2t) + C.$$
(3)
$$\int t^7\sin(2t^4)dt = \frac{1}{16}(\sin(2t^4) - 2t^4\cos(2t^4)) + C.$$
Hint: Use $u = 2t^4$ first then integrate by parts.

3. Partial fractions

(1) Integrate $\int \frac{x^3-2x^2}{x^4+5x^2+4}dx$.
(2) Integrate $\int \frac{x^6-1}{x^3+x^2-x-1}dx$.
(3) Integrate $\int \frac{x^4+2x^2+1}{x^3+2x^2+4x+8}dx$.

Answer:

(1)

$$\frac{x^3-2x^2}{x^4+5x^2+4} = \frac{2}{3}\frac{1}{1+x^2} - \frac{1}{3}\frac{x}{1+x^2} - \frac{8}{3}\frac{1}{4+x^2} + \frac{4}{3}\frac{x}{4+x^2}.$$

$$\int \frac{x^3-2x^2}{x^4+5x^2+4}dx = \frac{2}{3}\arctan x - \frac{1}{6}\ln 1+x^2 - \frac{4}{3}\arctan\frac{x}{2} + \frac{2}{3}\ln 4+x^2 + C.$$

(2)

$$\frac{x^6-1}{x^3+x^2-x-1} = -2+2x-x^2+x^3+\frac{3}{1+x}.$$

$$\int \frac{x^6-1}{x^3+x^2-x-1}dx = -2x+x^2-\frac{x^3}{3}+\frac{x^4}{4}+3\ln|1+x|+C.$$

(3)

$$\frac{x^4+2x^2+1}{x^3+2x^2+4x+8} = -2+x+\frac{25}{8}\frac{1}{2+x}+\frac{9}{4}\frac{1}{x^2+4}-\frac{9}{8}\frac{x}{x^2+4}.$$

$$\int \frac{x^4+2x^2+1}{x^3+2x^2+4x+8}dx = -2x+\frac{x^2}{2}+\frac{25}{8}\ln|2+x|+\frac{9}{8}\arctan\frac{x}{2}-\frac{9}{16}\ln(x^2+4)+C.$$

4. VOLUME OF REVOLUTION

Use both disk method and shell method to find the integral of the following solid of revolution:

(1) The area bounded by $xy = 10$, $x + y = 7$, rotating about $y = -2$.
(2) The area bounded by $x = y^2/4$, $y = 1$, $x = 5$, rotating about $x = -1$.
(3) (Hard) The area bounded by $y^2 = \ln x$, $x = e$, rotating about $x = -1$.

Answer:

(1) Disk method:
$$V = \pi \int_2^5 ((7 - x + 2)^2 - (10/x + 2)^2)dx = (51 - 40 \ln \frac{5}{2})\pi.$$
Shell method:
$$V = 2\pi \int_2^5 (y + 2)(7 - y - 10/y)dy = .$$
(2) Explained in review section
(3) Disk method:
$$V = \pi \int_{-1}^1 ((1 + e)^2 - (1 + e^{y^2})^2)dy$$