MATH 3B WORKSHEET 10

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1. Integrate by parts

(1) Integrate $\int_1^2 x^4(\ln x)^2 dx$.
(2) Integrate $\int e^{\sqrt{x}} dx$.

Answer:

(1) $\int_1^2 x^4(\ln x)^2 dx = \frac{62}{125} - \frac{64}{25}\ln 2 + \frac{32}{5}(\ln 2)^2$
(2) $\int e^{\sqrt{x}} dx = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C$.
Hint: Use u-sub first, let $u = \sqrt{x}$.

2. Partial fractions

Integrate $\int \frac{x^6-1}{x^4-x^3+2x^2}dx$.

$$\frac{x^6-1}{x^4-x^3+2x^2} = x^2+x-1-\frac{1}{4x}-\frac{1}{2x^2}-\frac{11}{4}\frac{x-1/2}{(x-1/2)^2+7/4}+\frac{7}{8}\frac{1}{(x-1/2)^2+7/4}$$

$$\int \frac{x^6-1}{x^4-x^3+2x^2}dx = \frac{x^3}{3}+\frac{x^2}{2}-x-\frac{1}{4}\ln|x|+\frac{1}{2x}-\frac{11}{8}\ln((x-1/2)^2+7/4)+\frac{\sqrt{7}}{4}\arctan\frac{2x-1}{\sqrt{7}}+C$$

3. VOLUME OF REVOLUTION

Use both disk method and shell method to find the integral of the following solid of revolution:
The area bounded by $x = y^2 - 1$, $y = (x+1)^3$, rotating about $y = 3$.
Answer: $15\pi/7$.
Disk method:

$$V = \pi \int_{-1}^{0} ((3 - (x+1)^3)^2 - (3 - \sqrt{x+1})^2) dx$$

Shell method:

$$V = 2\pi \int_{0}^{1} (3 - y)((\sqrt[3]{y} - 1) - (y^2 - 1)) dy$$