MATH 3B WORKSHEET 2

DANNING LU

1. Quick Review

(1) State the definition of Riemann sum.

(2) State the definition of net area.

2. Practice Problems

2.1. **Find the Riemann sum for arbitrary n.**

(1) $f(x) = x^2$, $0 \leqslant x \leqslant 3$.

(2) $f(x) = 3x - \frac{2}{x^2}$, $3 \leqslant x \leqslant 4$.

(3) $f(x) = \frac{2x}{x^2+1}$, $1 \leqslant x \leqslant 3$.

(4) $f(x) = x^2 + \sqrt{1+2x}$, $4 \leqslant x \leqslant 7$.

(5) $f(x) = \sqrt{\sin x}$, $0 \leqslant x \leqslant \pi$.

2.2. **Find the region whose area is evaluated by the given limit.**

(1) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{3n} \cdot \left(\frac{3i}{n}\right)^3$

(2) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$

(3) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{64n^2}{27i^3}$

(4) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan \frac{i\pi}{4n}$

2.3. **Use geometric results to evaluate the net area.**

(1) $y = 3x$, $a \leqslant x \leqslant b$, when
(a) $0 < a < b$:

(b) $a < 0 < b$:

(c) $a < b < 0$:

(2) $y = -\sqrt{1 - x^2}$:
(a) $-1 \leqslant x \leqslant 1$:

(b) $\frac{1}{2} \leqslant x \leqslant 1$:

(c) *$-1 \leqslant x \leqslant a$, where $-1 < a < 1$:

2.4. ***Estimate the deviation of Riemann sum.** Let A be the area under the graph of an increasing continuous function f from a to b, and let L_n and R_n be the approximations to A with n subintervals using left and right endpoints, respectively.

(1) How are A, L_n, and R_n related? Give a ranking for them for all the n's.

(2) Show that
$$R_n - L_n = \frac{b-a}{n}[f(b) - f(a)].$$
Try to explain the geometric meaning of this equation.

(3) Deduce that
$$R_n - A < \frac{b-a}{n}[f(b) - f(a)].$$

(4) If we want to find the area under the curve $y = 2^x$ from 1 to 3, use the above results to find a value n such that the Riemann sum has a maximum deviation of 0.0001.

3. Quizzes

NAME:______________ PERM:__________ SECTION TIME:______

Use left endpoints to estimate the net area under the curve $y = 5 - x^2$ between $x = -3$ and $x = 3$ by using 6 rectangles. Briefly explain the geometric meaning of the net area by using the graph of $y = 5 - x^2$ given below.