MATH 3B WORKSHEET 3 ANSWERS

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1. Quick Review

(1) What is definite integral? State the relations and differences among the following terminologies: *Area under the curve, Riemann sum, Definite integral.*

(2) State the *Fundamental Theorem of Calculus.*

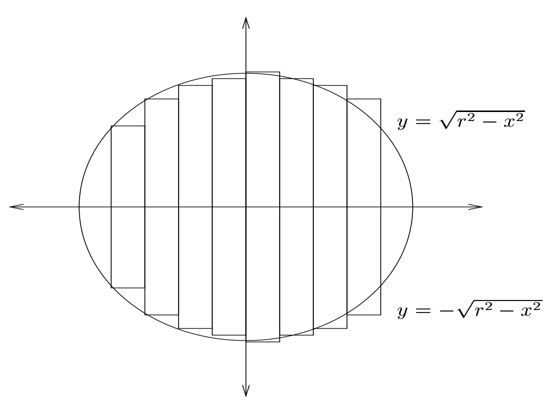
(a)

(b)

2. Discuss Problems (Multiple Choice Problems)

(1) Water is pouring out of a pipe at the rate of $f(t)$ gallons/minute. You collect the water that flows from the pipe between $t = 2$ and $t = 4$. The amount of water you collect can be represented by:
(a) $\int_2^4 f(x)dx$
(b) $f(4) - f(2)$
(c) $(4-2)f(4)$
(d) the average of $f(4)$ and $f(2)$ times the amount of time that elapsed
Answer: (a)

(2) Suppose we are going to consider the disk of radius r as the region bounded between the graphs of the functions $\sqrt{r^2 - x^2}$, and $-\sqrt{r^2 - x^2}$. Which of the following statements is true?



(a) The area of the region is given by the formula: $\int_{-r}^{r} 2\sqrt{r^2 - x^2}\, dx$

(b) The area of the disk can be written as a the limit of Riemann Sums of rectangles of length Δx and height $2\sqrt{r^2 - x_i^2}$ where the x_i are a partition of the interval $[-r, r]$.
(c) Both (a) and (b).
(d) The area cannot be found this way, because we cannot integrate the function $\sqrt{r^2 - x^2}$.

Answer: (c)

(3) A sprinter practices by running various distances back and forth in a straight line in a gym. Her velocity at t seconds is given by the function $v(t)$. What does $\int_0^{60} |v(t)| dt$ represent?
(a) The total distance the sprinter ran in one minute
(b) The sprinter's average velocity in one minute
(c) The sprinter's distance from the starting point after one minute
(d) None of the above

Answer: (a)

(4) Suppose f is a differentiable function. Then $\int_0^x f'(t)\, dt = f(x)$
(a) Always
(b) Sometimes
(c) Never

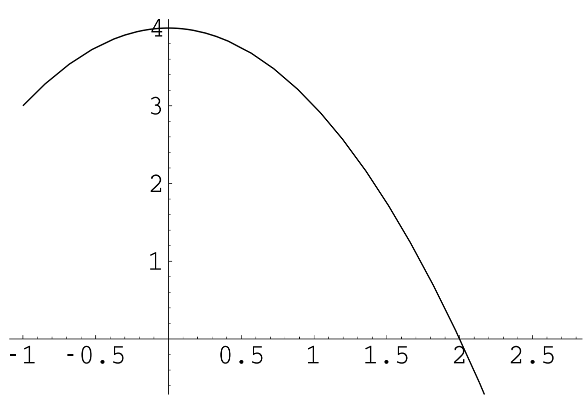
Justify your answer.

Answer: (b). From FTC(II) we know that $\int_0^x f'(t)\, dt = f(x) - f(0)$, *so we know that the question holds if and only if* $f(0) = 0$.

(5) Suppose the function $f(t)$ is continuous and always positive. If G is an antiderivative of f, then we know that G:
(a) is always positive.
(b) is sometimes positive and sometimes negative.
(c) is always increasing.
(d) There is not enough information to conclude any of the above.

Answer: (c). f *is the derivative of* G, *thus* $f > 0$ *implies* $G' > 0$, *and therefore* G *is increasing.*

(6) Below is the graph of a function f.



Let $g(x) = \int_0^x f(t)\, dt$. Then for $0 < x < 2$, $g(x)$ is
(a) increasing and concave up.

(b) increasing and concave down.
(c) decreasing and concave up.
(d) decreasing and concave down.
Answer: (b).
(a) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 0$
(b) $g(0) = 0$, $g'(0) = 4$ and $g'(2) = 0$
(c) $g(0) = 1$, $g'(0) = 0$ and $g'(2) = 1$
(d) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 1$
Answer: (b).

(7) You are traveling with velocity $v(t)$ that varies continuously over the interval $[a, b]$ and your position at time t is given by $s(t)$. Which of the following represent your average velocity for that time interval:
(a) $\dfrac{\int_a^b v(t)dt}{(b-a)}$
(b) $\dfrac{s(b)-s(a)}{b-a}$
(c) $v(c)$ for at least one c between a and b
Answer: (a), (b) and (c). (c) holds because of the Mean Value Theorem.

3. Practice Problem Sets

3.1. **Use FTC(I) to evaluate the derivatives of the following functions.**

(1) $g(x) = \int_0^x \sqrt{t+t^3}dt$.
By FTC(I), we know that $g'(x) = \sqrt{x+x^3}$.

(2) $g(x) = \int_1^x \ln(1+t^2)dt$.
By FTC(I), we know that $g'(x) = \ln(1+x^2)$.

(3) * $g(x) = \int_x^0 \sqrt{1+\sec t}dt$.
Since $g(x) = \int_x^0 \sqrt{1+\sec t}dt = -\int_0^x \sqrt{1+\sec t}dt$, by FTC(I) we know that $g'(x) = -\sqrt{1+\sec x}$.

(4) * $g(x) = \int_1^{e^x} \ln t dt$.
Let $h(u) = \int_1^u \ln t dt$, then by FTC(I) we have $h'(u) = \ln u$. And we have $g(x) = h(e^x)$. So by chain rule, $g'(x) = h'(e^x)\cdot(e^x)' = \ln(e^x)\cdot e^x = xe^x$.

(5) * $g(x) = \int_{\sin x}^{\sqrt{x}} \frac{z^2}{z^4+1}dz$.
Hint: $g(x) = \int_{\sin x}^{\sqrt{x}} \frac{z^2}{z^4+1}dz = \int_{\sin x}^{0} \frac{z^2}{z^4+1}dz + \int_0^{\sqrt{x}} \frac{z^2}{z^4+1}dz$.
Answer: $g'(x) = \dfrac{x}{x^2+1}\cdot\dfrac{1}{2\sqrt{x}} - \dfrac{\sin^2 x}{\sin^4 x+1}\cdot\cos x$.
Note: As practice, please write the steps for u-substitutions.

3.2. **Evaluate the integrals.**

(1) $\int_1^3 (x^2+2x-4)dx$.

$$\int_1^3 (x^2+2x-4)dx = [\frac{x^3}{3}+x^2-4x]|_1^3 = \frac{26}{3}.$$

(2) $\int_{-1}^1 x^{100}dx$.

$$\int_{-1}^1 x^{100}dx = [\frac{x^{101}}{101}]|_{-1}^1 = \frac{2}{101}.$$

(3) $\int_1^9 \sqrt{x}dx$.

$$\int_1^9 \sqrt{x}dx = [\frac{2}{3}x^{3/2}]|_1^9 = \frac{52}{3}.$$

(4) $\int_1^8 \frac{1}{\sqrt[3]{x}}dx$.

$$\int_1^8 \frac{1}{\sqrt[3]{x}}dx = [\frac{3}{2}x^{2/3}]|_1^8 = \frac{9}{2}.$$

(5) $\int_{\pi/6}^{\pi} \sin\theta d\theta$.

$$\int_{\pi/6}^{\pi} \sin\theta d\theta = [-\cos x]|_{\pi/6}^{\pi} = 1+\frac{\sqrt{3}}{2}.$$

(6) $\int_0^1 (u+2)(u-3)du$.

$$\int_0^1 (u+2)(u-3)du = \int_0^1 (u^2-u-6)du = [\frac{u^3}{3}-\frac{u^2}{2}-6u]|_0^1 = -\frac{37}{6}.$$

(7) $\int_0^4 (4-t)\sqrt{t}dt$.

$$\int_0^4 (4-t)\sqrt{t}dt = \int_0^4 (4t^{1/2}-t^{3/2})dt = [\frac{8}{3}t^{3/2}-\frac{2}{5}t^{5/2}]|_0^4 = \frac{128}{15}.$$

(8) $\int \frac{2+x^2}{\sqrt{x}}dx$.

$$\int \frac{2+x^2}{\sqrt{x}}dx = \int (2x^{-1/2}+x^{3/2})dx = 4\sqrt{x}+\frac{2}{5}x^{5/2}+C.$$

(9) $\int -\cos x dx$.

$$\int -\cos x dx = -\sin x + C.$$

(10) $\int (2\sin x - e^x)dx$.

$$\int (2\sin x - e^x)dx = -2\cos x - e^x + C.$$

(11) $\int 2^s ds$.

$$\int 2^s ds = \frac{1}{\ln 2}\cdot 2^s + C.$$