MATH 3B WORKSHEET 3

DANNING LU

1. Quick Review

(1) What is definite integral? State the relations and differences among the following terminologies: *Area under the curve, Riemann sum, Definite integral.*

(2) State the *Fundamental Theorem of Calculus.*

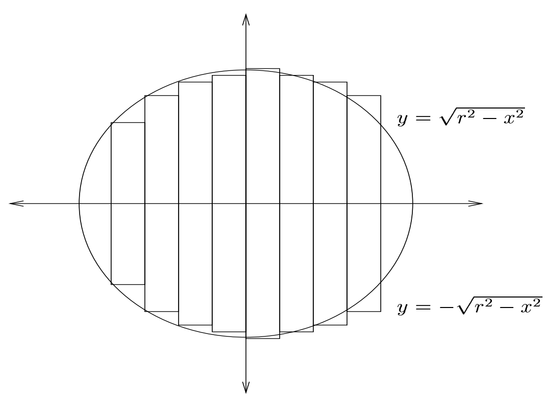
(a)

(b)

2. Discuss Problems (Multiple Choice Problems)

(1) Water is pouring out of a pipe at the rate of $f(t)$ gallons/minute. You collect the water that flows from the pipe between $t = 2$ and $t = 4$. The amount of water you collect can be represented by:
(a) $\int_2^4 f(x)dx$
(b) $f(4) - f(2)$
(c) $(4 - 2)f(4)$
(d) the average of $f(4)$ and $f(2)$ times the amount of time that elapsed

(2) Suppose we are going to consider the disk of radius r as the region bounded between the graphs of the functions $\sqrt{r^2 - x^2}$, and $-\sqrt{r^2 - x^2}$. Which of the following statements is true?



(a) The area of the region is given by the formula: $\int_{-r}^{r} 2\sqrt{r^2 - x^2}\, dx$
(b) The area of the disk can be written as a the limit of Riemann Sums of rectangles of length Δx and height $2\sqrt{r^2 - x_i^2}$ where the x_i are a partition of the interval $[-r, r]$.

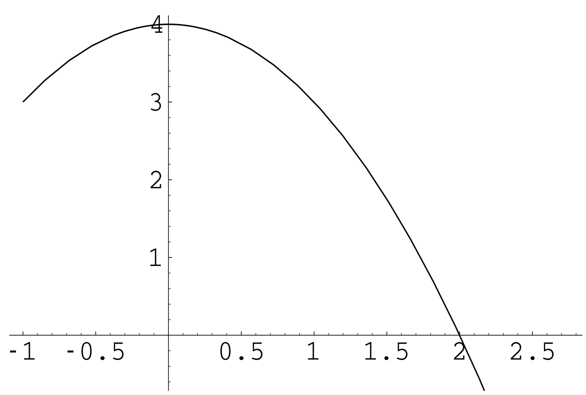
(c) Both (a) and (b).
(d) The area cannot be found this way, because we cannot integrate the function $\sqrt{r^2 - x^2}$.

(3) A sprinter practices by running various distances back and forth in a straight line in a gym. Her velocity at t seconds is given by the function $v(t)$. What does $\int_0^{60} |v(t)| dt$ represent?
(a) The total distance the sprinter ran in one minute
(b) The sprinter's average velocity in one minute
(c) The sprinter's distance from the starting point after one minute
(d) None of the above

(4) Suppose f is a differentiable function. Then $\displaystyle\int_0^x f'(t)\,dt = f(x)$
(a) Always
(b) Sometimes
(c) Never
Justify your answer.

(5) Suppose the function $f(t)$ is continuous and always positive. If G is an antiderivative of f, then we know that G:
(a) is always positive.
(b) is sometimes positive and sometimes negative.
(c) is always increasing.
(d) There is not enough information to conclude any of the above.

(6) Below is the graph of a function f.



Let $g(x) = \displaystyle\int_0^x f(t)\,dt$. Then for $0 < x < 2$, $g(x)$ is
(a) increasing and concave up.
(b) increasing and concave down.
(c) decreasing and concave up.
(d) decreasing and concave down.
(a) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 0$
(b) $g(0) = 0$, $g'(0) = 4$ and $g'(2) = 0$
(c) $g(0) = 1$, $g'(0) = 0$ and $g'(2) = 1$
(d) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 1$

(7) You are traveling with velocity $v(t)$ that varies continuously over the interval $[a, b]$ and your position at time t is given by $s(t)$. Which of the following represent your average velocity for that time interval:

(a) $\dfrac{\int_a^b v(t)dt}{(b-a)}$
(b) $\dfrac{s(b)-s(a)}{b-a}$
(c) $v(c)$ for at least one c between a and b

3. Practice Problem Sets

3.1. Use FTC(I) to evaluate the derivatives of the following functions.

(1) $g(x) = \int_0^x \sqrt{t+t^3}dt$.

(2) $g(x) = \int_1^x \ln(1+t^2)dt$.

(3) * $g(x) = \int_x^0 \sqrt{1+\sec t}dt$.

(4) * $g(x) = \int_1^{e^x} \ln t dt$.

(5) * $g(x) = \int_{\sin x}^{\sqrt{x}} \frac{z^2}{z^4+1}dz$.

3.2. **Evaluate the integrals.**

(1) $\int_1^3 (x^2 + 2x - 4)dx$.

(2) $\int_{-1}^1 x^{100} dx$.

(3) $\int_1^9 \sqrt{x} dx$.

(4) $\int_1^8 \frac{1}{\sqrt[3]{x}} dx$.

(5) $\int_{\pi/6}^{\pi} \sin\theta d\theta$.

(6) $\int_0^1 (u+2)(u-3)du$.

(7) $\int_0^4 (4-t)\sqrt{t}dt$.

(8) $\int \frac{2+x^2}{\sqrt{x}} dx$.

(9) $\int -\cos x dx$.

(10) $\int (2\sin x - e^x)dx$.

(11) $\int 2^s ds$.

4. Quizzes

NAME:______________ PERM:__________ SECTION TIME:______

Decide whether the following statements are True or False. Assume that f and g are continuous on the interval $[a, b]$.

(1) $\int_a^b f(x)dx$ is the area bounded by the graph of f, the x-axis and the lines $x = a$ and $x = b$.
(2) $\int_a^b f(x)dx$ is a number.
(3) $\int_a^b f(x)dx$ is an antiderivative of $f(x)$.
(4) $\int_a^b f(x)dx$ may not exist.
(5) If $\int f(x)dx = \int g(x)dx$, then $f(x) = g(x)$.
(6) If $f'(x) = g'(x)$, then $f(x) = g(x)$.
(7) If $f(x)$ is negative on $[a, b]$, then $\int_a^b f(x)dx$ might be zero.
(8) There exist two constants m and M such that $m(b-a) \leqslant \int_a^b f(x)dx \leqslant M(b-a)$.
(9) Since $(\sin x)' = \cos x$, we have $\int \cos x dy = \sin x$.
(10) Since $(\sin x)'' = -\sin x$, the antiderivative of $\int -\sin x dx$ is $\sin x + C$, where C is a constant.