

MATH 3B WORKSHEET 5

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1. INTEGRATE BY PARTS

1.1. Set I.

- (1) $\int (5x + 4)^{100} dx$
Use $u = 5x + 4$. Answer is $\frac{(5x+4)^{101}}{505} + C$.
- (2) $\int 3t^2(t^3 + 4)^5 dt$
Use $u = t^3 + 4$. Answer is $\frac{(t^3+4)^6}{6} + C$.
- (3) $\int \sqrt{4x - 5} dx$
Use $u = 4x - 5$ or $u = \sqrt{4x - 5}$. Answer is $\frac{(4x-5)^{3/2}}{6} + C$.
- (4) $\int \frac{2t^2}{\sqrt{t^3+4}} dt$
Use $u = t^3 + 4$. Answer is $\frac{4\sqrt{4+t^3}}{3} + C$.
- (5) $\int \cos(2x + 1) dx$
Use $u = 2x + 1$. Answer is $\frac{1}{2} \sin(2x + 1) + C$.
- (6) $\int \sin^9 t \cos t dt$
Use $u = \sin t$. Answer is $\frac{\sin^{10} t}{10} + C$.
- (7) $\int x e^{x^2} dx$
Use $u = x^2$. Answer is $\frac{e^{x^2}}{2} + C$.

1.2. Set II.

- (1) $\int \tan x \sec^4 x dx$
Use $u = \sec x$ or $u = \cos x$. Answer is $\sec^4 x + C$.
- (2) $\int \frac{(\sqrt{x}-1)^2}{\sqrt{x}} dx$
Use $u = \sqrt{x} - 1$. Answer is $\frac{2}{3}(\sqrt{x} - 1)^3 + C$.
- (3) $\int \sqrt{x^3 + x^2}(3x^2 + 2x) dx$
Use $u = x^3 + x^2$. Answer is $\frac{2}{3}(x^3 + x^2)^{3/2} + C$.
- (4) $\int_{-1}^1 \frac{x+1}{(x^2+2x+3)^3} dx$
Use $u = x^2 + 2x + 3$. Answer is $\frac{1}{18}$.
- (5) $\int_0^{\pi/2} \sin x \sqrt{\cos x} dx$
Use $u = \cos x$. Answer is $\frac{2}{3}$.
- (6) $\int (x + 1) \sin(x^2 + 2x + 3) dx$
Use $u = x^2 + 2x + 3$. Answer is $-\frac{1}{2} \cos(x^2 + 2x + 3) + C$.

(7) $\int \left(1 + \frac{1}{t}\right)^3 \frac{1}{t^2} dt$
 Use $u = 1 + \frac{1}{t}$. Answer is $-\frac{1}{4}\left(1 + \frac{1}{t}\right)^4 + C$.

1.3. Set III.

- (1) $\int_{-1}^1 x^2 \sqrt{x^3 + 1} dx$
 Use $u = x^3 + 1$. Answer is $\frac{4\sqrt{2}}{9}$.
- (2) $\int \frac{2}{\sqrt{3x-7}} dx$
 Use $u = 3x - 7$ or $u = \sqrt{3x - 7}$. Answer is $\frac{4}{3}\sqrt{3x - 7} + C$.
- (3) $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x}+1)^2} dx$
 Use $u = \sqrt{x} + 1$. Answer is $\frac{1}{3}$.
- (4) $\int_0^1 \frac{x}{\sqrt{x+1}} dx$
 Use $u = \sqrt{x + 1}$. Answer is $\frac{4-2\sqrt{2}}{3}$.
- (5) $\int x\sqrt{2x+1} dx$
 Use $u = \sqrt{2x + 1}$. Answer is $\frac{1}{15}(1 + 2x)^{3/2}(3x - 1) + C$.
- (6) $\int \sqrt{x}\sqrt{x}\sqrt{x+1} dx$
 Use $u = x\sqrt{x} + 1$. Answer is $\frac{4}{9}(1 + x\sqrt{x})^{3/2} + C$.
- (7) $\int_1^4 \frac{dx}{1+\sqrt{x}}$
 Use $u = 1 + \sqrt{x}$. Answer is $2 + 2\ln 2 - 2\ln 3$.

1.4. Set IV.

- (1) $\int x^3 \sqrt{x^2 + 1} dx$
 Use $u = \sqrt{x^2 + 1}$. Answer is $\frac{1}{15}(1 + x^2)^{3/2}(3x^2 - 2) + C$.
- (2) $\int (x^2 + 1)\sqrt{x - 2} dx$
 Use $u = \sqrt{x - 2}$. Answer is $\frac{2}{105}(x - 2)^{3/2}(67 + 24x + 15x^2) + C$.
- (3) $\int \frac{x^2+2x}{x^2+2x+1} dx$
 Use $u = x + 1$ and notice that $\frac{x^2+2x}{x^2+2x+1} = 1 - \frac{1}{u^2}$, we know that answer is $x + \frac{1}{1+x} + C$.
- (4) $\int \frac{dx}{x^2+6x+9}$
 Use $u = x + 3$ and notice that $u^2 = x^2 + 6x + 9$, we know that answer is $-\frac{1}{x+3} + C$.
- (5) $\int \frac{dx}{\cos^2 x (1 + \tan x)^3}$
 Use $u = 1 + \tan x$. Answer is $-\frac{1}{2(1 + \tan x)^2} + C$.
- (6) $\int x \tan(x^2) \sec^5(x^2) dx$
 Use $u = x^2$ then use $v = \sec u$ or $v = \sin u$. Answer is $\frac{1}{10} \sec^5(x^2) + C$.

1.5. *Set V.

- (1) $\int (\tan(2x) + \cot(2x))^2 dx$
 Hint: expand it. Answer is $\frac{\tan(2x) - \cot(2x)}{2} + C$.
- (2) $\int_0^a \sqrt{a^2 - x^2} dx$
 Use $u = a \sin x$. Answer is $\frac{a^2\pi}{4}$.

(3) $\int_0^\pi \sqrt{\sin^3 x - \sin^5 x} dx$

Use $u = \sin x$ and be careful of the signs. Answer is $\frac{4}{5}$.

(4) $\int \frac{dx}{x \ln x \ln(\ln x)}$

Use $u = \ln(\ln x)$. Answer is $\ln |\ln(\ln x)| + C$.

(5) $\int_0^3 \frac{x^2}{(x^2 - 2x + 2)^2} dx$

Use $x - 1 = \tan u$. Answer is $\frac{3}{10} + \frac{\pi}{4} + \arctan 2$.

(6) $\int_0^\pi (1 - \sin^3 \theta) d\theta$

Use $u = \cos \theta$. Answer is $\pi - \frac{3}{4}$.

(7) $\int_0^1 (x - x^2) \sqrt{1 - x^2} dx$

Hint: Use $x = \sin u$. You need to use the double angle formula. Answer is

$$\frac{1}{3} - \frac{\pi}{16}.$$

(8) $\int_0^\pi e^{\sin^2 x} \cos x dx$

Use $u = \sin x$. Answer is 0.