MATH 3B WORKSHEET 6 ANSWER

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1. Area between Curves

1.1. **Quick Review.** Draw a picture illustrating area between two curves, and write the formula of which you are going to use in order to evaluate the area.

1.2. **Exercises: Find the areas.**

(1) The area bounded by $y = \sqrt[3]{x}$, $y = 1/x$ and $x = 8$.
Answer: $Area = \int_1^8 (\sqrt[3]{x} - 1/x)dx = \frac{45}{4} - \ln 8$.

(2) The area bounded by $y = \sqrt{2x+6}$, $y = -\sqrt{2x+6}$, $y = x - 1$.
Answer: It's easier to integrate along y-axis. By $y = \pm\sqrt{2x+6}$ we get $x = \frac{y^2-6}{2}$. By solving $\begin{cases} y^2 = 2x+6 \\ y = x-1 \end{cases}$ we get $y = -2$ and $y = 4$. Hence $Area = \int_{-2}^4 ((1+y) - \frac{y^2-6}{2})dy = 18$.

(3) The area bounded by $x = 1 - y^2$, $x = y^2 - 1$.
Answer: This one is also easier if you integrate along y-axis. $Area = \int_{-1}^1 ((1 - y^2 - (y^2 - 1))dy = \frac{8}{3}$.

(4) The area bounded by $y = \frac{1}{4}x^2$, $y = 2x^2$, $x + y = 3$, where $x \geqslant 0$.
Answer: You need to separate the intervals with $0 < x < 1$ and $1 < x < 2$. $Area = \int_0^1 (2x^2 - \frac{1}{4}x^2)dx + \int_1^2 ((3-x) - \frac{1}{4}x^2)dx = \frac{3}{2}$.

2. Finding Volume with Disk Method

2.1. Quick Review. Draw a picture illustrating the volume of which we are evaluating by using disk method, and write the formula of which you are going to use in order to evaluate the volume.

2.2. Exercises: Find the volumes.

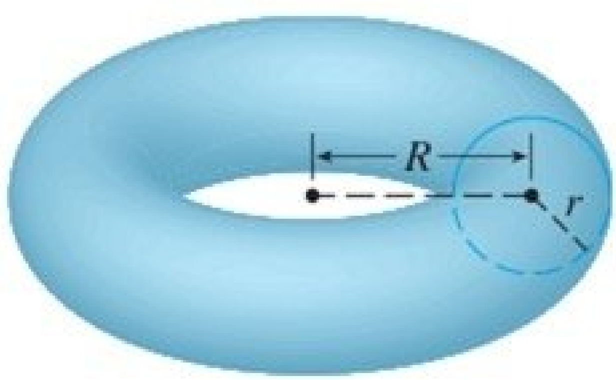
(1) The solid obtained by rotating the region bounded by $y = \sqrt{x-1}$, $y = 0$, $x = 5$ about the x-axis.
Answer: $Volume = \int_1^5 \pi(\sqrt{x-1})^2 dx = 8\pi$.

(2) The solid obtained by rotating the region bounded by $y = x$, $y = \sqrt[4]{x}$ about the x-axis.
Answer: $Volume = \int_0^1 (\pi(\sqrt[4]{x})^2 - \pi x^2)dx = \pi/3$.

(3) The solid obtained by rotating the region bounded by $y = x$, $y = \sqrt[4]{x}$ about the y-axis.
Answer: We need to integrate it along y-axis, so we change $y = \sqrt[4]{x}$ into $x = y^4$. $Volume = \int_0^1 \pi(y^2 - (y^4)^2)dy = 2\pi/9$.

(4) *The solid obtained by rotating the region bounded by $xy = 1$, $y = 0$, $x = 1$, $x = 2$ about the line $x = -1$.
Answer: We are integrating along the y-axis. Do notice that the radii is $x + 1$ instead of x. Hence $Volume = \int_0^{1/2} \pi((2+1)^2 - (1+1)^2 dy + \int_{1/2}^1 \pi((1/y+1)^2 - (1+1)^2)dy = (2 + \ln 4)\pi$.

(5) The torus as shown in the graph.



Explained in section.