

MATH 3B WORKSHEET 9

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1. QUICK REVIEW

Steps for integrating partial fractions:

- (1) Do long division if degree of numerator is larger than denominator.
- (2) Fully factorize the denominator and complete the square for quadric terms.
- (3) Use method of undetermined coefficient to split the fraction. What are the general terms?
- (4) What's the integrate of the following terms?

$$\int \frac{1}{x+a} dx, \int \frac{1}{(x+a)^k} dx, \int \frac{1}{(x+a)^2+c} dx, \int \frac{x+a}{(x+a)^2+c} dx$$

2. PRACTICE PROBLEMS

2.1. Set I.

- (1) $\int \frac{x^3}{x+3} dx$
- (2) $\int \frac{2x+3}{x^2+3x-10} dx$
- (3) $\int \frac{x+1}{x^2-2x+5} dx$
- (4) $\int \frac{1}{x(x^2+1)} dx$
- (5) $\int \frac{3}{x^3+1} dx$
- (6) $\int \frac{x^2+1}{(x+1)^2(x-1)} dx$

2.2. Set II.

- (1) $\int \frac{x}{(x+1)(x+2)(x+3)} dx$
- (2) $\int \frac{x^5+x^4-8}{x^3-x} dx$
- (3) $\int \frac{1}{(x^2+1)(x^2+x)} dx$
- (4) $\int \frac{1}{x^4-1} dx$
- (5) $\int \frac{(x+1)^2}{(x^2+1)^2} dx$
- (6) $\int \frac{-x^2-2}{(x^2+x+1)^2} dx$

2.3. Set III.

- (1) $\int \frac{1}{1+\sqrt[3]{x+1}} dx$
- (2) $\int \frac{(\sqrt{x})^3-1}{\sqrt{x+1}} dx$
- (3) $\int \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} dx$
- (4) $\int \frac{1}{\sqrt{x+\sqrt{x}}} dx$
- (5) $\int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x}$
- (6) $\int \frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}} dx$

2.4. *Reading material and Set IV. (Not required)

In this section, we are going to deal with rational trigonometric functions. We are taking the advantage of the fact that all the six basic trigonometric functions can be expressed as rational functions of $\tan \frac{x}{2}$, namely,

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}.$$

Hence we do the substitution $u = \tan \frac{x}{2}$. Hence we can get that $x = 2 \arctan u$ and so $dx = \frac{2}{1+u^2} du$. Thus we can get a partial fraction after the substitution, which we already know how to deal with.

- (1) $\int \frac{1}{3+\cos x} dx$
- (2) $\int \frac{1}{3+\sin^2 x} dx$
- (3) $\int \frac{1}{2+\sin x} dx$
- (4) $\int \frac{1}{1+\sin x+\cos x} dx$
- (5) $\int \frac{1}{2 \sin x - \cos x + 5} dx$
- (6) $\int 3 \sin^2 x \cos^3 x dx$

3. QUIZZES

NAME:----- PERM:----- SECTION TIME:-----

Integrate

$$\int \frac{1}{(x^2 + 1)(x^2 + x + 1)} dx$$