10 For lecture on 11/2

1. State which of the following sets are connected and/or simply-connected.
   (a) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \neq 1\}$;
   (b) $\mathbb{R}^3 - \{(x, y, z) : x^2 + y^2 = 1, z = 1\}$;
   (c) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 > 1\}$;
   (d) $\mathbb{R}^3 - \{(\cos t, \sin t, t) : 0 \leq t \leq \pi\}$;
   (e) $\mathbb{R}^3 - \{(\cos t, \sin t, t) : t \in \mathbb{R}\}$;
   (f) $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, z < 1\}$.
2. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = (y^2 \cos x, 2y \sin x)$, C is any path starting at (1,1) and ending at (1,3).
3. Determine whether $\mathbf{F}$ is a gradient vector field, and if so, find a function f such that $\mathbf{F} = \nabla f$.
   (a) $\mathbf{F} = (e^x \sin y + x^2, e^x \cos y + y^2)$;
   (b) $\mathbf{F} = (4x^2 - 4y^2 + x, 7xy + \ln y)$;
   (c) $\mathbf{F} = (xy^2 + 3x^2 \ln x + x^2, x^3/y)$;
   (d) $\mathbf{F} = (yz + e^x \sin z, xz + y^2 - e^y, xy + e^x \cos z)$;
   (e) $\mathbf{F} = (y \cos(xy), x \cos(xy) - z \sin y, \cos y)$.
4. Let $\mathbf{F} = (-y/(x^2 + y^2), x/(x^2 + y^2), 1$.
   (a) Check that $\nabla \times \mathbf{F} = \mathbf{0}$ in its domain.
   (b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by $\mathbf{r}(t) = (\cos t, \sin t, 0)$, with $0 \leq t \leq \pi$.
   (c) Based on your calculation, is $\mathbf{F}$ a gradient vector field? Explain your judgement.