3 For Lecture on 10/3

1. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$.
   (a) $f(x, y) = x^y + y \sin x$
   (b) $f(x, y) = xe^{x^2+2xy-y^2}$
   (c) $f(x, y) = e^{xy} \cos x \ln(y - x^2)$
   (d) $f(x, y) = \arctan(x/y)$
2. Find the gradient for $f(x, y, z) = (y^2 + 1)e^{xyz}$.
3. Find the Jacobian for $F(x, y) = (y, -x \sin y, e^x)$.
4. Find the directional derivative of $f(x, y) = 3x^2 - 2xy$ in the direction of $(3, 4)$ at point (1,3).
5. Find linear approximation of $f(x, y) = e^{xy} - xy^3$ at point (1,0).
6. Find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 = 1$ at the point (x_0, y_0, z_0). For extra practice, do this in two ways, one by solving for z and consider the two cases $z_0 \geq 0$ and $z_0 < 0$, and the other by viewing the sphere as a level surface.
7. A hill has the shape of the graph $z = f(x, y) = 40 - x^2 - 2y^2$.
   (a) A ball is held at (1,2,31) on the hill. When it is released, it will move in the direction of the steepest descend. Give a unit vector $\vec{u}$ (in 3 dimension) in the direction of the ball when it is released.
   (b) A hiker is also at point (1,2,31), and wants to hike uphill. However, the hiker can not go in any paths that have slope bigger than 1. Assume that the angle between the direction of the path (in 2d, projected onto the xy-plane) and ∇f is θ. Give the range for θ.
8. Estimate $\cos(0.01)e^{0.02}$.
9. Use differentials to estimate the amount of tin in a closed tin can with diameter 3 inch and height 4 inch if the top and bottom is 0.02 inch thick and the side is 0.01 inch thick.
10. Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S but you know that the curves

$$r_1(t) = (2 + 3t, 1 - t^2, 3 - 4t + t^2)$$

$$r_2(t) = (1 + t^2, 2t^3 - 1, 2t + 1)$$

both lie on S. Find an equation of the tangent plane at P.