6 For lecture on 10/15

1. Find the maximal and minimal of the functions in the given region.
   (a) $f(x, y) = x^2 + xy + y^2 + y$, $-1 \le x \le 1$, $-1 \le y \le 1$.
   (b) $f(x, y) = e^x \cos y$, $-1 \le x \le 0$, $-\pi/2 \le y \le 5\pi/2$.
   (c) $f(x, y) = (x - y)(1 - xy)$, $0 \le x \le 2$, $0 \le y \le 2$.

2. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2x$ on the set D, which is the closed triangular region with vertices $(2, 0)$, $(0, 2)$ and $(0, -2)$.

3. Use Lagrange multipliers to find the extreme values of the function subject to the given constraint.
   (a) $f(x, y) = x^2 - y^2$; $x^2 + y^2 = 1$.
   (b) $f(x, y) = xe^y$; $x^2 + 2y^2 = 2$.
   (c) $f(x, y, z) = \ln(x^2 + 1) + \ln(y^2 + 1) + \ln(z^2 + 1)$; $x^2 + y^2 + z^2 = 12$.

4. Given function $f(x, y) = x^2 + y^2 + 4x - 4y$ and the region D given by $x^2 + y^2 \le 9$. Find the maximum and minimum of the function on the region D.

5. Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin. (As an exercise, use two methods to solve this question. You can view z as a function of x and y, or you can use the Lagrange Multipliers.)

6. *Find the points on both the plane $x + y + 2z = 2$ and the paraboloid $z = x^2 + y^2$ that are nearest to and farthest from the origin.

7. **Find the maximum and minimum values of

$$f(x, y, z) = ye^{x-z}$$

   subject to the restrictions $9x^2 + 4y^2 + 36z^2 = 36$ and $xy + yz = 1$.

8. ***
   (a) Find the maximum value of

$$f(x_1, ..., x_n) = \sqrt[n]{x_1 x_2 ... x_n}$$

   given that $x_1, x_2, ..., x_n$ are positive numbers and $x_1 + x_2 + ... + x_n = c$, where c is a constant.

   (b) Deduce from part (a) that if $x_1, x_2, ..., x_n$ are positive numbers, then

$$\sqrt[n]{x_1 x_2 ... x_n} \le \frac{x_1 + x_2 + ... + x_n}{n}.$$

   Under what circumstances the equality holds?