MATH 6A 19F with Prof. Pan
(Answers and selected solutions)

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Office Hour: W 10:00-11:00, South Hall 6432G
http://web.math.ucsb.edu/~danninglu/teaching/math_6A_F19/math_6A_F19.html

1 For lecture on 9/26

1. $(2, 2, -2)$.
2. $\sqrt{8}$.
3. 14.
4. $\arccos(-\frac{1}{\sqrt{10}})$.
5. $a = \frac{5}{4}$.
6. $(-3, -16, 7)$.
7. $\sqrt{306}$.
8. $\frac{1}{2}\sqrt{230}$.
9. (a) $\overrightarrow{u} \cdot (\overrightarrow{v} + \overrightarrow{w}) = 3$.
   (b) $\overrightarrow{u} \cdot (\overrightarrow{v} \times \overrightarrow{w}) = 52$.
   (c) $\overrightarrow{u} \times (\overrightarrow{v} + \overrightarrow{w}) = \langle 11, 9, 3\rangle$.
   (d) $(\overrightarrow{u} \cdot \overrightarrow{v})\overrightarrow{w}\langle 28, 42, 14\rangle$.
10. $\langle x(t), y(t), z(t)\rangle = \langle 3 - 2t, 4 - 3t, 5 - 4t\rangle$.
11. $\langle x(t), y(t), z(t)\rangle = \langle 2 - t, 5 - 6t, -3 + 4t\rangle$ $(0 \le t \le 1)$.
12. $-2x - 3y - z = -11$.
13. $-5x + 13y + 6z = 39$.

2 For lecture on 10/1

1. Find the domain for the following functions.

   (a) $\{(x, y) : x \geq 2y,\ x \neq 0\}$.

   (b) $\{(x, y) : x \geq 2y, x > 0$ or $x \leq 2y, x < 0\}$.

   (c) $\{(x, y) : x^2 + y^2 < 16\}$.

2. Find the limit of $f(x, y)$ as $(x, y) \to (0, 0)$, if it exists; or state the reason if it does not exist:

   (a) Limit exist and equals to 0, since denominator is not zero.

   (b) Limit does not exist.

   (c) Limit does not exist.

   (d) Limit does not exist. (Hint: Choose $y = 0$ and $x = y^3$.)

   (e) Limit exist and equals to 0. (Hint: Let $u = 2x^2 + y^2$.)

   (f) Limit exist and equals to 0. (Hint: Squeeze theorem, or just use polar coordinate.)

3. $f \circ g(u, v) = (v + 2u, \sqrt{u + v} + 3v)$.
   Domain: $\{(u, v) : u + v \geq 0\}$.
   $g \circ f(x, y, z) = (z + 3x, -x + 2y, \sqrt{4x - 2y + z})$.
   Domain: $\{(x, y, z) : 4x - 2y + z \geq 0\}$.

3 For Lecture on 10/3

1. (a) $\frac{\partial f}{\partial x} = yx^{y-1} + y\cos x$.
   $\frac{\partial f}{\partial y} = x^y \ln(x) + \sin x$.

   (b) $\frac{\partial f}{\partial x} = (2x^2 + 2xy - 1)e^{x^2+2xy-y^2}$.
   $\frac{\partial f}{\partial y} = (2x^2 - 2xy)e^{x^2+2xy-y^2}$.

   (c) $\frac{\partial f}{\partial x} = ye^{xy}\cos(x)\ln(y - x^2) + e^{xy}\sin(x)\ln(y - x^2) - e^{xy}\cos x\frac{2x}{y-x^2}$.
   $\frac{\partial f}{\partial y} = xe^{xy}\cos(x)\ln(y - x^2) + e^{xy}\cos(x)\frac{1}{y-x^2}$.

   (d) $\frac{\partial f}{\partial x} = \frac{1}{y} \cdot \frac{1}{1+(x/y)^2}.//\frac{\partial f}{\partial y} = -\frac{x}{y^2} \cdot \frac{1}{1+(x/y)^2}$.

2. $\nabla f = (yz(y^2 + 1)e^{xyz}, (2y + xz(y^2 + 1))e^{xyz}, xy(y^2 + 1)e^{xyz})$.

3. $DF = \begin{bmatrix} 0 & 1 \\ -\sin y & -x\cos y \\ e^x & 0 \end{bmatrix}$.

4. -8/5.

5. $L_{(1,0)}(x, y) = 1 + y$.

6. $x_0x + y_0y + z_0z = 1$. Solution:

   (a) Method 1: Assume that $z_0 > 0$. (It will be similar if $z_0 < 0$.) Then we can write $z = \sqrt{1 - x^2 - y^2}$. Let $f(x, y) = \sqrt{1 - x^2 - y^2}$, we have $f_x = \frac{-x}{\sqrt{1-x^2-y^2}}$ and $f_y = \frac{-y}{\sqrt{1-x^2-y^2}}$. Hence the linear approximation is $dz = -\frac{x_0}{\sqrt{1-x_0^2-y_0^2}}dx - \frac{y_0}{\sqrt{1-x_0^2-y_0^2}}dy$. So the equation for the tangent plane is

   $$z - z_0 = -\frac{x_0}{\sqrt{1 - x_0^2 - y_0^2}}(x - x_0) - \frac{y_0}{\sqrt{1 - x_0^2 - y_0^2}}(y - y_0).$$

   (b) Method 2: Let $G(x, y, z) = x^2 + y^2 + z^2$. Then the sphere is the level surface $G = 1$. By the properties of gradient, we know that $\mathbf{n} = \nabla G(x_0, y_0, z_0) = \langle 2x_0, 2y_0, 2z_0\rangle$ is a normal vector for the tangent plane. Hence the equation for the tangent plane can be expressed by $\mathbf{n} \cdot (\langle x, y, z\rangle - \langle x_0, y_0, z_0\rangle) = 0$, i.e.,

   $$2x_0(x - x_0) + 2y_0(y - y_0) + 2z_0(z - z_0) = 0.$$

7. (a) Solution: The height decreases most rapidly along $u = -\frac{\nabla f(1,2)}{||\nabla f(1,2)||} = (\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}})$, and the rate of descend is $-||\nabla f(1, 2)|| = -2\sqrt{17}$. Hence the ball is moving in the direction of $(\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}, -2\sqrt{17})$. Now we just need to make it into a unit vector.
   Answer: $\frac{1}{\sqrt{69}}(\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}, -2\sqrt{17})$.

(b) Solution: We have $\frac{\partial f}{\partial u} = ||\nabla f|| \cos\theta$, and so $0 < 2\sqrt{17}\cos\theta \leq 1$.
Answer: $\arccos\frac{1}{2\sqrt{17}} \leq \theta < 1$.

8. Answer: Let $f(x, y) = \cos x e^y$. Then $f_x = -\sin x e^y$ and $f_y = \cos x e^y$. The linear approximate at $(0, 0)$ is

$$f(\Delta x, \Delta y) \approx 1 + 0 \cdot (\Delta x - 0) + 1 \cdot (\Delta y - 0).$$

Make $\Delta x = 0.01$ and $\Delta y = 0.02$ and we get $\cos(0.01)e^{0.02} \approx 1.02$.

9. Answer: We know that the volume of the tin can is expressed by $V = \pi r^2 h$, where r and h denote the radius and height, respectively. Hence we know by linear approximation,

$$dV = (2\pi rh)dr + (\pi r^2)dh.$$

By substituting $r = 1.5$, $h = 4$, $dr = 0.01$, $dh = 0.02 \times 2 = 0.04$ we get volume of tin is $dV = 0.21\pi\,\mathrm{in}^3$.

10. Answer: Since $r_1(0) = r_2(1) = (2, 1, 3)$, and $r_1'(0) = \langle 3, 0, -4\rangle$, $r_2'(1) = \langle 2, 6, 2\rangle$, we know that these two vectors all lie in the tangent plane. Thus we can get a normal vector

$$n = \langle 3, 0, -4\rangle \times \langle 2, 6, 2\rangle = \langle 24, -14, 18\rangle.$$

So the equation for the tangent plane is $24(x-2)-14(y-1)+18(z-3) = 0$, which simplifies to $12x - 7y + 9z = 44$.

4 For Lecture on 10/8

1. $DF = \begin{pmatrix} y & x & 0 \\ 0 & z\cos y & \sin y. \end{pmatrix}$. $DG = \begin{pmatrix} 1 & 1 \\ 2u & 0 \\ 0 & e^v \end{pmatrix}$. $D(F \circ G) = \begin{pmatrix} y + 2xu & y \\ 2uz\cos y & e^v \sin y \end{pmatrix}$.

2. Answer: When $(s,t) = (1,0)$, we have $x = 1$, $y = 1$ and $z = 0$. $\frac{\partial u}{\partial x}(1,1,0) = 4$, $\frac{\partial u}{\partial y}(1,1,0) = 1$ and $\frac{\partial u}{\partial z}(1,1,0) = 0$. $u(1,1,0) = 1$

   $$\frac{\partial u}{\partial s}(1,0) = \frac{\partial u}{\partial x}(1,1,0)\frac{\partial x}{\partial s}(1,0)+\frac{\partial u}{\partial y}(1,1,0)\frac{\partial y}{\partial s}(1,0)+\frac{\partial u}{\partial z}(1,1,0)\frac{\partial z}{\partial s}(1,0) = 4{\cdot}1+1{\cdot}2+0{\cdot}0 = 6$$

   $$\frac{\partial u}{\partial t}(1,0) = \frac{\partial u}{\partial x}(1,1,0)\frac{\partial x}{\partial t}(1,0)+\frac{\partial u}{\partial y}(1,1,0)\frac{\partial y}{\partial t}(1,0)+\frac{\partial u}{\partial z}(1,1,0)\frac{\partial z}{\partial t}(1,0) = 4{\cdot}1+1{\cdot}(-1)+0{\cdot}0 = 3.$$

   So the tangent plane is given by $u = 1+6(s-1)+3(t-0)$, or $u = 6s+3t-5$.

3. Answer: Since we have $v \cdot v = ||v||^2$ for any vector v, magnitude of $c(t)$ remains constant indicates that

   $$c(t) \cdot c(t) = P$$

   for some constant P. By taking derivative on both sides, we get that

   $$c(t) \cdot c'(t) = 0,$$

   which implies that $c(t)$ is perpendicular to $c'(t)$.

5 No problem set for Lecture on 10/10

6 For lecture on 10/15

1. (a) Global maximum=4, at $(x, y) = (1, 1)$.
   Global minimum=-1/3, at $(x, y) = (1/3, -2/3)$.

   (b) Global maximum=1, at $(x, y) = (0, 0)$ or $(0, 2\pi)$.
   Global minimum=-1, at $(x, y) = (0, \pi)$.

   (c) Global maximum=2, at $(x, y) = (2, 0)$.
   Global minimum=-2, at $(x, y) = (0, 2)$.

2. Global maximum=4, at $(x, y) = (0, 2)$ or (0,-2).
   Global minimum=-1, at $(x, y) = (1, 0)$.

3. (a) Global maximum=1, at $(x, y) = (-1, 0)$ or (1,0).
   Global minimum=-1, at $(x, y) = (0, 1)$ or (0,-1).

   (b) Global maximum=$\sqrt{\sqrt{5}-1}e^{\frac{\sqrt{5}-1}{2}}$, at $(x, y) = (\sqrt{\sqrt{5}-1}, \frac{\sqrt{5}-1}{2})$.
   Global minimum=$-\sqrt{\sqrt{5}-1}e^{\frac{\sqrt{5}-1}{2}}$, at $(x, y) = (-\sqrt{\sqrt{5}-1}, \frac{\sqrt{5}-1}{2})$.

   (c) Global maximum=$3\ln(5)$, at $(x, y, z) = (\pm 2, \pm 2, \pm 2)$ (8 points).
   Global minimum=ln(13), at $(x, y, z) = (\pm 2\sqrt{3}, 0, 0)$ or switch between x, y and z (6 points).

4. Global maximum=$9 + 12\sqrt{2}$, at $(x, y) = (3/\sqrt{2}, -3/\sqrt{2})$.
   Global minimum=-8, at $(x, y) = (-2, 2)$.

5. $(0, \pm 3, 0)$

6. Solution: Let $f(x, y, z) = x^2 + y^2 + z^2$, $g_1(x, y, z) = x + y + 2z - 2$ and $g_2(x, y, z) = z - x^2 - y^2$. Lagrange multiplier gives us $\nabla f - \lambda \nabla g_1 - \mu \nabla g_2 = 0$, which further gives us

$$\begin{cases} 2x - \lambda + 2x\mu = 0 \\ 2y - \lambda + 2y\mu = 0 \\ 2z - 2\lambda - \mu = 0 \\ x + y + 2z = 2 \\ z = x^2 + y^2 \end{cases}$$

   From the first two equation, we get $\lambda = 2x(1 + \mu) = 2y(1 + \mu)$. Hence $x = y$ or $\mu = -1$.
   If $\mu = -1$, then $\lambda = 0$, and thus $z = -1$ from the third equation. But $z = x^2 + y^2 \geq 0$, which leads to contradiction.

   Thus $x = y$, and so from the last equation we get $z = 2x^2$, and from the fourth equation we get $2x + 2z = 2$. Combine these two equations we get $4x^2 + 2x - 2 = 0$, and so $x = 1/2$ or $x = -1$.

So we have two candidates: (1/2,1/2,1/2) and (-1,-1,2). Compare these two, we get:

Minimal distance=$\sqrt{3}/2$ at point (1/2,1/2,1/2).

7. Solution: Let $g_1(x,y,z) = 9x^2+4y^2+36z^2-36$ and $g_2(x,y,z) = xy+yz-1$. Lagrange multiplier gives us $\nabla f - \lambda\nabla g_1 - \mu\nabla g_2 = 0$, which further gives us

$$\begin{cases} ye^{x-z} - 18\lambda x - \mu y = 0 \\ e^{x-z} - 8\lambda y - \mu(x+z) = 0 \\ -e^{x-z} - 72\lambda z - \mu y = 0 \\ 9x^2 + 4y^2 + 36z^2 = 36 \\ xy + yz = 1 \end{cases}$$

Seems like we need to use calculator to solve this equation. I should not put this question here.

8. ***

(a) We only need to consider $g(x_1, ..., x_n) = x_1...x_n$, with restriction $h(x_1, ..., x_n) = x_1 + ... + x_n - c$. By Lagrange multiplier, we have

$$\frac{\partial g}{\partial x_i} = \frac{x_1...x_n}{x_i} = \lambda\frac{\partial h}{\partial x_i} = 1$$

for all $i = 1, ..., n$. Thus

$$x_1...x_n = \lambda x_1 = \lambda x_2 = ... = \lambda x_n.$$

Since $x_1, ..., x_n$ all positive, we know that $\lambda \neq 0$, and so $x_1 = x_2 = ... = x_n$. From $h = 0$, we know that $x_1 = ... = x_n = c/n$. Thus the extreme value of f is $f(c/n, ..., c/n) = \sqrt[n]{(c/n)^n} = c/n$.

(b) From part (a), we know that $f(x_1, ..., x_n) \leq c/n = \frac{x_1+x_2+...+x_n}{n}$. The result follows.

7 For lecture on 10/17

1. Let $\vec{F}(x, y, z) = (\sin x, \cos z, \tan y)$ be a vector field. Find the following:

   (a) $\nabla \cdot \vec{F} = \cos x$.

   (b) $\nabla \times \vec{F} = (\sec^2 y + \sin z, 0, 0)$.

   (c) $DF = \begin{pmatrix} \cos x & 0 & 0 \\ 0 & 0 & -\sin z \\ 0 & \sec^2 y & 0 \end{pmatrix}$.

   (d) If $\vec{u}(t)$ is a curve in 3-dimension space, with $\vec{u}(0) = (1, 2, 3)$ and $\vec{u}'(0) = (4, 5, 6)$, find $(\vec{F} \circ \vec{u})'(0) = (4 \sin 1, -6 \cos 3, 5 \sec^2 2)$.

2. $div\, F = e^x \sin y$.
   $curl\, F = e^x \cos y - \sin x$.

3. When $(x, y) = (1, 1)$, it is a local minimum.
   When $(x, y) = (-1, -1)$, it is a local minimum.
   When $(x, y) = (0, 0)$, it is a saddle point.

4. Solution: Denote $A(-2, 3)$, $B(5, 3)$, $C(5, -4)$.

   First of all, we can consider

$$g(x, y) = x^3 - 48x + y^3 - 12y$$

   instead.

   First we find critical points: $\nabla g = (3x^2 - 48, 3y^2 - 12) = 0$, which gives us 4 solutions: (4,2), (4,-2), (-4,2), (-4,-2). However, only the first two points are inside the region. Thus we get two candidates:
   $g(4, 2) = -144$
   $g(4, 2) = -112$.

   Now consider boundary.
   On AB, the line can be given by $y = 3$.
   $g(x, 3) = x^3 - 48x - 9 = h_1(x)$.
   $0 = h_1'(x) = 3x^2 - 48$ gives $x = 4$ or $x = -4$. But only $x = 4$ is on the line segment. Thus we get one candidate:
   $g(4, 3) = -137$.

   On BC, the line can be given by $x = 5$.
   $g(5, y) = -115 + y^3 - 12y = h_2(y)$.
   $0 = h_2'(y) = 3y^2 - 12$ gives $y = 2$ or $y = -2$. Both of them are candidates:
   $g(5, 2) = -99$.
   $g(5, -2) = -131$.

On AC, the line can be given by $y = 1 - x$.
$g(x, 1-x) = 3x^2 - 39x - 11 = h_3(x)$.
$0 = h_1'(x) = 6x - 39$ gives $x = 13/2$, which is not on the line segment. Thus we don't have candidates.

We also get three vertices as candidates:
Point A: $g(-2, 3) = 97$.
Point B: $g(5, 3) = -124$.
Point C: $g(5, -4) = -131$.

Compare all candidates, we get the minimal of g occurs at $g(4, 2) = -144$, and the maximal occurs at $g(-2, 3) = 97$. Thus the maximal and minimal of f are $\sqrt[3]{97}$ and $-\sqrt[3]{144}$, respectively.

8 For lecture on 10/23

1. (reference Midterm practice question 5)Let γ be the graph of $y = e^{-x^2}.x \in [-2, 2]$ in $\mathbb{R}^2$.

   (a) $\gamma(t) = (t, e^{-t^2})$, $t \in [-2, 2]$.

   (b) $L(\gamma) = \int_{-2}^{2} \sqrt{1 + (-2te^{-t^2})^2} dt$

   (c) $S(u, v) = (u, e^{-u^2} \cos v, e^{-u^2} \sin v)$, $u \in [-2, 2].v \in [0, 2\pi]$.

   (d) We have $T_u = (1, -2ue^{-u^2} \cos v, -2ue^{-u^2} \sin v)$ and $T_v = (0, -e^{-u^2} \sin v, e^{-u^2} \cos v)$. The point corresponds to $(u, v) = (1, 5\pi/3)$.
   $N(1, 5\pi/3) = T_u(1, 5\pi/3) \times T_v(1, 5\pi/3) = (1, -1/e, \sqrt{3}/e) \times (0, \frac{\sqrt{3}}{2e}, \frac{1}{2e})$
   $= (-\frac{1}{e^2}, -\frac{1}{2e}, \frac{\sqrt{3}}{2e})$.
   Thus the tangent plane is given by $(-\frac{1}{e^2}, -\frac{1}{2e}, \frac{\sqrt{3}}{2e}) \cdot (x - 1, y - \frac{1}{2e}, z + \frac{\sqrt{3}}{2e}) = 0$, which simplifies to
   $-\frac{1}{e^2}x - \frac{1}{2e}y + \frac{\sqrt{3}}{2e}z = -\frac{2}{e^2}$

2. $S(u, v) = (2 + 2u - v, 1 - u - v, 7 - 2u - 6v)$, $0 \leq u \leq 1$, $0 \leq v \leq 1 - u$.

9 For lecture on 10/31

1. (a) $-14\sqrt{53}\pi^2$.
   (b) 0.
   (c) $1/96(-8 + 222\sqrt{5} - 3\sinh^{-1}(2))$.
   (d) 0.
   (e) 6π.
2. (a) -2.
   (b) $11/2$.
   (c) $e^4/2 - 19/6$.
   (d) $15/2$.
   (e) -2π.
3. * $-2\sin 3$. Hint: this is a gradient vector field.

10 For lecture on 11/5

1. (a) Not connected;
   (b) Connected but not simply-connected;
   (c) Simply-connected;
   (d) Simply-connected;
   (e) Connected but not simply-connected;
   (f) Simply-connected.
2. $8 \sin 1$.
3. (a) $f = e^x \sin y + x^3/3 + y^3/3)$;
   (b) Not gradient vector field;
   (c) Not gradient vector field;
   (d) $f = xyz + e^x \sin z + y^3/3 - e^y$;
   (e) $f = \sin(xy) + z \cos y$.
4. (a) Just check.
   (b) 2π.
   (c) It is not a gradient vector field, since the domain is not simply-connected.

11 For lecture on 11/7

1. Evaluate the integrals.

   (a) 16/3.

   (b) 16.

   (c) $\int_0^6 \int_0^{1-x/6} xydydx = \int_0^1 \int_0^{6-6y} xydxdy = 3/2$.

2. 91/3.

3. (a) $\int_0^1 \int_{y/2}^{y} e^{y-x}dxdy + \int_1^2 \int_{y/2}^{1} e^{y-x}dxdy$.

   (b) $\int_0^2 \int_0^{\sqrt{6x}} dydx + \int_2^4 \int_0^{\sqrt{16-x^2}} dydx$.

   (c) $\int_0^7 \int_y^{3+\sqrt{y+9}} f(x,y)dxdy + \int_{-9}^0 \int_{3-\sqrt{y+9}}^{3+\sqrt{y+9}} f(x,y)dxdy$.

   (d) $\int_1^8 \int_{\sqrt[3]{y}}^{y} f(x,y)dxdy$.

4. $$\int_0^7 \int_0^{x/7} e^{x^2} dydx = \frac{e^{49}-1}{14}.$$

12 For lecture on 11/12

1. $\int_1^2 \int_{y-1}^{7-3y} xydxdy = 13/3$.
2. Find Volume of solid.
   (a) $\int_0^{7/3} \int_0^{\frac{7-3x}{2}} (7-3x-2y)dydx = 343/36$.
   (b) $\int_0^2 \int_0^{y/2} (6-5x^2)dxdy = 31/6$.
   (c) $\int_0^{2\pi} \int_0^{\sqrt{2}} (\sqrt{3-r^2} - r^2/2)rdrd\theta = (2\sqrt{3} - 5/3)\pi$.

13 For lecture on 11/14

1. Convert from Cartesian coordinate to polar coordinate before integration.

   (a) $\int_0^{\pi/4} \int_0^1 r^3 \sin\theta \cos\theta dr d\theta = 1/16$

   (b) $\int_0^{2\pi} \int_0^2 (4 - r^2) r dr d\theta = 8\pi$.

   (c) $\int_0^{2\pi} \int_3^4 \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r dz dr d\theta = 28\sqrt{7}\pi/3$.

2. $\int_1^2 \int_{y-1}^{7-3y} xy dx dy = 13/3$.

3. $\int_0^{\pi/4} \int_1^2 \theta r dr d\theta = 3\pi^2/64$.

4. $\int_0^{2\pi} \int_0^1 (3r\cos\theta)(4r\sin\theta)(12r) dr d\theta = 0$.

14 For lecture on 11/19

1. Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

   (a) $\int_0^2 \int_0^1 \int_0^3 (xy + z^2) dzdydx = 21.$

   (b) $\int_0^3 \int_0^x \int_{x-y}^{x+y} y dzdydx = 27/2.$

   (c) $\int_0^{\pi} \int_0^{\pi-x} \int_0^x \sin y \, dzdydx = \frac{\pi^2-4}{2}.$

   (d) $f = \int_0^2 \int_{-1}^1 \int_{x^2-1}^{1-x^2} (x-y) dzdxdy = -16/3.$

   (e) $\int_0^3 \int_0^{1-x/3} \int_0^{2-2y-2x/3} xz \, dzdydx = 3/10.$

2. Skipped.

3. Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

   (a) $\int_0^{2\pi} \int_0^4 \int_{-5}^4 r^2 dzdrd\theta = 384\pi.$

   (b) $\int_0^{2\pi} \int_0^2 \int_{r^2}^4 zrdzdrd\theta = 64\pi/3.$

   (c) $\int_0^{\pi/2} \int_0^2 \int_0^{4-r^2} (r\cos\theta + r\sin\theta + z) rdzdrd\theta = 128/15 + 8\pi/3.$

4. Skipped.

5. Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

   (a) $\int_0^{2\pi} \int_0^{\pi/3} \int_0^1 (\rho\sin\phi\sin\theta)^2 (\rho\cos\phi)^2 (\rho^2\sin\phi) d\rho d\phi d\theta = 47\pi/3360.$

   (b) $\int_0^{2\pi} \int_0^{\pi} \int_1^{2\sqrt{2}} \rho\sin\phi\cos\theta e^{\rho^2} \rho^2 \sin\phi d\rho d\phi d\theta = 0.$

6. Use change of variables $\begin{cases} x = u + 2w \\ y = 1 - u - v \\ z = u + 5v + 7w \end{cases}.$

$$\int_0^1 \int_0^{1-u} \int_0^{1-u-v} ((u+2w) - 2(1-u-v) + 3(u+5v+7w)) 15 dwdvdu = 95/4$$

15 For lecture on 11/21

Evaluate the surface integral $\iint_S f\,dS$.

1. Let $x = t + 2s$, $y = 1 - t + 2s$, $z = t$.
   $\iint_S f\,dS = \int_0^1 \int_0^{1-t} (t+2s)(1-t+2s)t \cdot \sqrt{24} ds dt = \frac{11}{5\sqrt{6}}$.

2. Let $x = r\cos\theta$, $y = r\sin\theta$, $z = \sqrt{3}r$.
   $\iint_S f\,dS = \int_0^{2\pi} \int_1^2 3r^2 \cdot 2r dr d\theta = 45\pi$.

3. Let $x = x$, $y = \cos\theta$, $z = \sin\theta$.
   $\iint_S f\,dS = \int_0^{2\pi} \int_1^2 (2\sin\theta - \cos\theta + x^2) \cdot 1 dx d\theta = 14\pi/3$.

4. Let $x = \cos\theta\sin\phi$, $y = \sin\theta\sin\phi$, $z = \cos\phi$.
   $\iint_S f\,dS = \int_0^{\pi/2} \int_0^{\pi/2} (\sin^2\phi\sin\theta\cos\theta + 3\cos\phi) \cdot \sin^2\phi d\phi d\theta = 19\pi/32$.

5. 49π.

6. Let $x = x$, $y = e^{-x^2}\cos\theta$, $z = e^{-x^2}\sin\theta$.
   $\iint_S f\,dS = \int_0^1 \int_{\arcsin(e^{x^2-1})}^{\pi/2} \frac{xe^{-x^2}\cos\theta}{(e^{-x^2})^2\sqrt{4x^2e^{-2x^2}+1}} \cdot e^{-x^2}\sqrt{4x^2e^{-2x^2}+1} d\theta dx =$ $1/2e$.

16 For lecture on 12/3

1. Let $x = u$, $y = v$, $z = 2u^2v$. $\iint_S \vec{F} \cdot d\vec{S} = -\int_0^1 \int_0^2 (u^2v, uv^3, 4u^3v^2) \cdot (-4uv, -2u^2, 1) dv du = 2$.

2. $-\pi$.