MATH 6A WORKSHEET 1 ANSWERS

DANNING LU

1. Things to Announce

My name: Danning Lu
My email address: danning.lu@math.ucsb.edu
My office hour: Thursdays 12.30-1.30 or by appointment
Section webpage:
http://web.math.ucsb.edu/~danninglu/teaching/math_6A_W19/math_6A_W19.html.
Section worksheets and answers will update periodically on this page. You may want to add a bookmark.
Where is my office? South Hall 6432G (pink side)
How to get a good score? Are there any resources?
You are supposed to attend all lectures and sections, and really focus. If you have any questions, feel free to go to my office hours, or the instructor's office hours. Also, mathlab and CLAS are also good resources. My math lab shift are Mondays and Wednesdays 4-5.

Attendance policy:
If you have a schedule conflict or an emergency, please email me in advance (at least email me before class begins), and we will figure out a solution. Basically, if you have a schedule conflict, it would be much easier if you can attend either of the other sections, which are on Tuesdays 8am, 4pm, 5pm, 6pm, 7pm, and on Wednesday 8am. The contents are the same for each sections. Please do NOT switch sections on GOLD, as if you dropped from GOLD we can not add you back.
We will probably have quizzes each week except for today.

2. Problems

(1) Find the vector pointing from $(0, 2, 3)$ to $(2, 4, 1)$.
    Answer: $\langle 2, 2, -2\rangle$.
(2) Find the vector pointing from $(7, -5, 9)$ to $(-1, 4, -8)$.
    Answer: $\langle -8, 9, -17\rangle$.
(3) Find the distance between the two points $(1, 2, 1)$ and $(3, 4, -1)$.
    Answer: $2\sqrt{3}$.
(4) Find the distance between the two points $(0, 3, 4)$ and $(5, 0, 0)$.
    Answer: $5\sqrt{2}$.
(5) Let $u =< 3, 2, 5 >$, $v =< 4, -1, -4 >$, $w =< 3, 3, -7 >$. Calculate
    (a) $u + v$.
    (b) $u - w$.
    (c) $3u + 7v - 2w$.

Answer:
(a) $\langle 7, 1, 1\rangle$.
(b) $\langle 0, -1, 12\rangle$.
(c) $\langle 31, -7, 1\rangle$.

(6) Assume that v is a vector in $\mathbb{R}^3$ and α is a positive number. Proof that the length of αv equals to the length of v multiplied by α.

Answer: Let $v = \langle x, y, z\rangle$. Then the length of αv is given by $\sqrt{(\alpha x)^2 + (\alpha y)^2 + (\alpha z)^2} = \alpha\sqrt{x^2 + y^2 + z^2}$, which equals to the length of v multiplied by α.

(7) Assume that v is a vector in $\mathbb{R}^2$ with length 0. Prove that v is the zero vector.

Answer: Let $v = \langle x, y, z\rangle$. Then we know that $0 = \sqrt{x^2 + y^2 + z^2}$, which means $0 = x^2 + y^2 + z^2$. However we know that $x^2 + y^2 + z^2 \leqslant 0$, and the equality holds if and only if $x = y = z = 0$. Hence we must have $x = y = z = 0$, which means that v is a zero vector.

(8) Find the Cartesian coordinates of points whose polar coordinates are $(0, \pi/2)$, $(4, \pi/2)$, $(6, 7\pi/4)$, $(3, 5\pi/6)$, $(12, 3\pi/2)$.

Answer: $(0, 0)$, $(0, 4)$, $(3\sqrt{2}, -3\sqrt{2})$, $(-3\sqrt{3}/2, 3/2)$, $(0, -12)$.

(9) Find the polar coordinates of points whose Cartesian coordinates are $(4, 4)$, $(-3, -\sqrt{3})$, $(2, -2\sqrt{3})$, $(0, \pi)$.

Answer: $(4\sqrt{2}, \pi/4)$, $(2\sqrt{3}, 7\pi/6)$, $(4, 5\pi/3)$, $(\pi, \pi/2)$.

(10) Assume $A(x, y, z)$ and $B(x', y', z')$ are two points in $\mathbb{R}^3$. Use the proposition of vectors to prove that all the points on the straight line AB has coordinate $(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y', \lambda z + (1-\lambda)z')$, where λ is some real number.

Proof: Assume that P is on the line AB, then we must have $\overrightarrow{BP} = \lambda\overrightarrow{BA}$ for some scalar λ. By calculation we know that $\overrightarrow{BA} = \langle x - x', y - y', z - z'\rangle$. Hence we know that $\overrightarrow{BP} = \langle \lambda(x - x'), \lambda(y - y'), \lambda(z - z')\rangle$. By further calculation we can know that P has coordinate $(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y', \lambda z + (1-\lambda)z')$.

(11) Use the above result to determine if the following points lies on the line segment AB, where $A(3, 5, 0)$ and $B(0, 0, 4)$.
(a) $(1, 2, 8/3)$.
(b) $(2, 10/3, 4/3)$.
(c) $(-3, -5, 8)$.

Answer:
(a) No.
(b) Yes. $\lambda = 2/3$.
(c) No. It is on the line, however $\lambda = -1$, making it to be on the extension line of AB.

(12) *Let $A(3, 0, 0)$ and $B(-3, 0, 0)$. Find the equation of the points P in $\mathbb{R}^3$ such that the sum of the lengths of the line segments AP and BP is 10.

Answer: $\sqrt{(x-3)^2 + y^2 + z^2} + \sqrt{(x+3)^2 + y^2 + z^2} = 10$, which you can simplify to the equation $x^2/25 + y^2/16 + z^2/16 = 1$.