MATH 6A WORKSHEET 2

DANNING LU

Note: I will use gradescope (www.gradescope.com) to grade your quizzes. You need to sign in with your ucsb student email (umail.ucsb.edu). The course entry code is 9XXB37 in case you are not automatically enrolled in gradescope.

1. Lecture Review

(1) How to calculate dot product?

(2) What is the geometric meaning of dot product?

(3) How to calculate cross product?

(4) What is the geometric meaning of cross product?

(5) Is dot product commutative/associative/distributive? What about the cross product?

(6) How do you find the vector/parametric equation of a line l if:
   (a) given a point $A = (a_1, a_2, a_3)$ on l and a vector $\overrightarrow{v} = \langle v_1, v_2, v_3 \rangle$ parallel to l.

   (b) given two points $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$ on the line.

(7) How do you find the equation of a plane p if:
   (a) given a point $A = (a_1, a_2, a_3)$ on p and a vector $\overrightarrow{n} = \langle n_1, n_2, n_3 \rangle \perp p$.

   (b) given three points $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$, $C = (c_1, c_2, c_3)$ on p, if they are not on the same line.

(8) What's the domain and range of a function of several variables?

2. Practice Problems

2.1. Dot Products and Cross Products.

(1) Find the dot product between the vectors $\langle 1,2,3\rangle$ and $\langle 4,-5,6\rangle$
Answer: 12.
(2) Find the dot product between the vectors $\langle 3,3,-1\rangle$ and $\langle -2,8,2\rangle$
Answer: 16.
(3) What is the angle between the vectors $\langle 1,0,-2\rangle$ and $\langle 3,5,4\rangle$?
Answer:=arccos $-\frac{1}{\sqrt{10}}$.
(4) What is the angle between the vectors $\langle 3,3,-1\rangle$ and $\langle -2,8,2\rangle$?
Answer: $\arccos\frac{16}{3\sqrt{38}}$.
(5) Find a real number a such that the vectors $\langle 3,a,4\rangle$ and $\langle a-3,1,1\rangle$ are perpendicular to each other.
Answer: Need $0=\langle 3,a,4\rangle\cdot\langle a-3,1,1\rangle=3(a-3)+a+4=4a-5$, hence $a=5/4$.
(6) Let $\alpha=\langle 3,-1,-1\rangle$ and $\beta=\langle 1,2,5\rangle$ be vectors. Find $\alpha\times\beta$.
Answer: $\alpha\times\beta=\langle -3,-16,7\rangle$.
(7) Find the area of the parallelogram with vertexes $(0,0,0)$, $(1,3,5)$, $(-2,-2,1)$ and $(-1,1,6)$.
Answer: $\langle 1,3,5\rangle\times\langle -2,-2,1\rangle=\langle 13,-11,4\rangle$, the length of which is $3\sqrt{34}$, which is the area.
(8) Find the area of the triangle with vertexes $(1,2,3)$, $(4,5,-1)$ and $(0,3,0)$.
Answer: $\frac{1}{2}\sqrt{230}$.
(9) Let $\overrightarrow{u}=\langle -3,3,2\rangle$, $\overrightarrow{v}=\langle -2,-4,2\rangle$, and $\overrightarrow{w}=\langle 2,3,1\rangle$. Find
  (a) $\overrightarrow{u}\cdot(\overrightarrow{v}+\overrightarrow{w})$.
  (b) $\overrightarrow{u}\cdot(\overrightarrow{v}\times\overrightarrow{w})$.
  (c) $\overrightarrow{u}\times(\overrightarrow{v}+\overrightarrow{w})$.
  (d) $(\overrightarrow{u}\cdot\overrightarrow{v})\overrightarrow{w}$.
Answer:
  (a) 3.
  (b) 52.
  (c) $\langle 11,9,3\rangle$
  (d) $\langle -4,-6,-2\rangle$

2.2. Equation of lines and planes.

(1) Find the equation of the line passing points $(3,4,5)$ and $(1,1,1)$.
Answer: Vector equation $\langle x,y,z\rangle=\langle 1,1,1\rangle+t\langle 2,3,4\rangle$, or parametrized equation
$$\begin{cases} x=1+2t \\ y=1+3t \\ z=1+4t \end{cases}.$$
(2) Find the equation of the line segment with end points $(2,5,-3)$ and $(1,-1,1)$.
Answer: Vector equation $\langle x,y,z\rangle=\langle 1,-1,1\rangle+t\langle 1,6,-4\rangle$, or parametrized equation $\begin{cases} x=1+t \\ y=-1+6t \\ z=1-4t \end{cases}$, both requiring $0\leqslant t\leqslant 1$.

(3) Find the plane that passes through the point $(1, 2, 3)$ and perpendicular with the vector $\langle -2, -3, -1\rangle$.
Answer: $2x + 3y - z = 11$.
(4) Find the plane that passes through the points $(1, 2, 3)$, $(4, 5, -1)$ and $(0, 3, 0)$.
Answer: $5x - 13y - 6z = -39$.

2.3. **Functions of several variables.** State the domain and range for the following functions:

(1) $f(x, y) = \frac{y}{x}$.
(2) $f(x, y) = \sqrt{x^2 + y}$.
(3) $f(x, y, z) = \frac{3}{x^2+y^2+z^2}$.

Answer:

(1) Domain: $\{(x, y) : x \neq 0\}$. Range: $(-\infty, \infty)$.
(2) Domain: $\{(x, y) : y \geqslant -x^2\}$. Range: $[0, \infty)$.
(3) Domain: $\{(x, y, z) : x, y, z$ not all equal to $0\}$. Range: $(0, \infty)$.

Quizzes

NAME:________________________
PERM:_________________________

Show your work. Partial points might be awarded. No calculators. You DO need to simplify your answers.

Assume that $\overrightarrow{u} = \langle 7, -1, -2\rangle$, $\overrightarrow{v} = \langle -4, 3, 2\rangle$ and $\overrightarrow{w} = \langle -3, -1, 1\rangle$.

(1) Find the length of $2\overrightarrow{v} - 3\overrightarrow{w}$.
Answer: $2\overrightarrow{v} - 3\overrightarrow{w} = \langle 1, 9, 1\rangle$, and hence length of $2\overrightarrow{v} - 3\overrightarrow{w}$ is $\sqrt{1^2 + 9^2 + 1^2} = \sqrt{83}$.

(2) Is $\overrightarrow{u} + 2\overrightarrow{v}$ perpendicular to $\overrightarrow{w}$? State your reason.
Answer: $\overrightarrow{u} + 2\overrightarrow{v} \cdot \overrightarrow{w} = 0$, hence perpendicular.