MATH 6A WORKSHEET 3

DANNING LU

1. Limit and Continuity

1. Find the limit of $f(x, y)$ as $(x, y) \to (0, 0)$, if it exists; or state the reason if it does not exist:

(1) $f(x, y) = \frac{x^3y - xy^3 - x}{1 - xy}$
(2) $f(x, y) = \frac{xy}{(x^2+y^2)^{3/2}}$
(3) $f(x, y) = \frac{\sin(3x - 2y + xy)}{3x - 2y + xy}$
(4) $f(x, y) = \frac{2x^2 - y^2}{x^2 + 2y^2}$
(5) $f(x, y) = (2x - y)e^{\frac{1}{y-2x}}$
(6) $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$
(7) $f(x, y) = \frac{\sin(3x^2+y^2)}{x^2+2y^2}$

2. Find all points where the vector valued function

$$\mathbf{F}(x,y,z) = \left(\frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}\right)$$

is not continuous.

3*. Let $\mathbf{x_0}$ be a point in $\mathbb{R}^3$, and $\mathbf{a}$ be a vector in $\mathbb{R}^3$. Find all points in $\mathbb{R}^3$ such that the following two vector valued functions $\mathbb{R}^3 \to \mathbb{R}^3$ is continuous, respectively.

$$\mathbf{F}(\mathbf{x}) = \frac{\mathbf{x}-\mathbf{x_0}}{||\mathbf{x}-\mathbf{x_0}||}$$

$$\mathbf{G}(\mathbf{x}) = \frac{\mathbf{x}\times\mathbf{a}}{||\mathbf{x}\times\mathbf{a}||}$$

4**. Let $f(x,y)$ be a function such that $\lim_{t\to 0} f(\lambda t, \mu t) = f(0,0)$ for any real numbers λ, μ. Is this sufficient to say that $f(x,y)$ is continuous at point $(0,0)$?

2. Partial Derivatives

1. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial}{\partial y}(\frac{\partial f}{\partial x})$, $\frac{\partial}{\partial x}(\frac{\partial f}{\partial y})$ for each of the following functions:

   (1) $f(x, y) = x^y + y \sin x$
   (2) $f(x, y) = xe^{x^2+2xy-y^2}$
   (3) $f(x, y) = e^{xy} \cos x \ln(y - x^2)$
   (4) $f(x, y) = \arctan(x/y)$

What do you discover?