MATH 6A WORKSHEET 4

DANNING LU

(1) Find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 = 1$ at the point (x_0, y_0, z_0). For extra practice, do this in two ways, one by solving for z and consider the two cases $z_0 \geq 0$ and $z_0 < 0$, and the other by viewing the sphere as a level surface.

(2) A soccer player is running in the field, where his position is described by the following equation:

$$r(t) = (16 \sin^3 t, 13 \cos t - 5 \cos(2t)).$$

Find the velocity and acceleration of the soccer player.

(3) A skier is on the mountain with equation

$$h = 100 - 0.4y^2 - 0.3x^2,$$

where h denotes the height.

(a) The skier is located at the point with xy-coordinates $(1, 1)$, and wants to ski downhill along the steepest possible path. In which direction (indicated by a vector (a, b) in the xy-plane) should the skier begin skiing?

(b) The skier begins skiing in the direction given by the xy-vector (a, b) you found in part (**??**) so the skier heads in a direction in space given by the vector (a, b, c). Find the value of c.

(c) A hiker located at the same point on the mountain decides to begin hiking downhill in a direction given by a vector in the xyplane that makes an angle θ with the vector (a, b) you found in part (**??**). How big should θ be if the hiker wants to head downhill along a path whose slope is at most 0.5 (in absolute value)?

(4) Estimate $\cos(0.01)e^{0.02}$.

(5) Use differentials to estimate the amount of tin in a closed tin can with diameter 3 inch and height 4 inch if the top and bottom is 0.02 inch thick and the side is 0.01 inch thick.

(6) The gas law for a fixed mass m of an ideal gas at absolute temperature T, pressure p and volume V is given below:

$$pV = nRT,$$

where R is a constant. Show that

$$\frac{\partial p}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial p} = -1.$$

(7) Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S but you know that the curves

$$r_1(t) = (2 + 3t, 1 - t^2, 3 - 4t + t^2)$$
$$r_2(t) = (1 + u^2, 2u^3 - 1, 2u + 1)$$

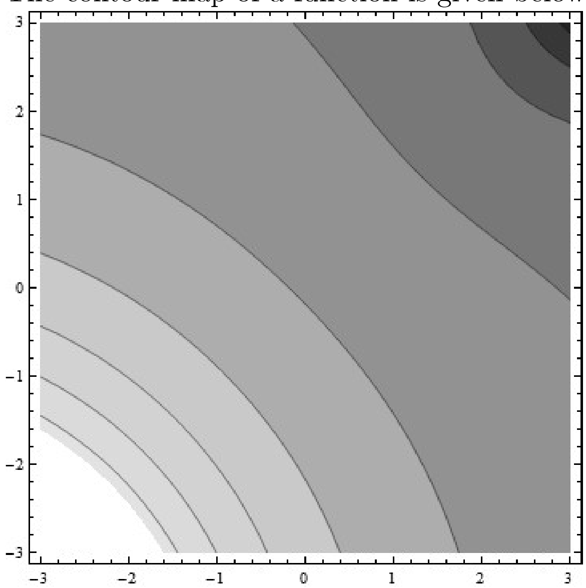
both lie on S. Find an equation of the tangent plane at P.

(8) Find the tangent plane of the function $u(s, t)$ given by $u = x^4y + y^2z^3$ and $x = se^t$, $y = s^2e^{-t}$ and $z = s \sin t$ at point $(s, t) = (1, 0)$.

(9) (a) Write down the transition function $\mathbf{\Phi}$ from spherical coordinate to Cartesian coordinate. (i.e., this is a function that you input the spherical coordinate and get Cartesian coordinate)
(b) Find the derivative $D\mathbf{\Phi}$.
(c) A curve has the form $(1 + 2t, \pi/2 - t, \pi + 3t)$ in spherical coordinate. Find the starting point of the curve when $t = 0$ in Cartesian coordinate, and find the velocity vector at $t = 0$ expressed in Cartesian coordinate.

(d) Do the same for cylindrical coordinate.

(10) The contour map of a function is given below.



Draw the gradient at point (-2,0). Estimate the partial derivatives at the same point. (Assume that the difference between adjacent lines is 1, and dark shadow represent lower value.)