MATH 6A WORKSHEET 6

DANNING LU

(1) Find the maximal and minimal of the functions in the given region.

(a) $f(x, y) = x^2 + xy + y^2 + y$, $-1 \leqslant x \leqslant 1$, $-1 \leqslant y \leqslant 1$.

(b) $f(x, y) = e^x \cos y$, $-1 \leqslant x \leqslant 0$, $-\pi/2 \leqslant y \leqslant 5\pi/2$.

(c) $f(x, y) = (x - y)(1 - xy)$, $0 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 2$.

(2) Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2x$ on the set D, which is the closed triangular region with vertices $(2, 0)$, $(0, 2)$ and $(0, -2)$.

(3) Use Lagrange multipliers to find the extreme values of the function subject to the given constraint.

(a) $f(x, y) = x^2 - y^2$; $x^2 + y^2 = 1$.

(b) $f(x, y) = xe^y$; $x^2 + 2y^2 = 2$.

(c) $f(x, y, z) = \ln(x^2 + 1) + \ln(y^2 + 1) + \ln(z^2 + 1)$; $x^2 + y^2 + z^2 = 12$.

(4) Given function $f(x, y) = x^2+y^2+4x-4y$ and the region D given by $x^2+y^2 \leqslant 9$. Find the maximum and minimum of the function on the region D.

(5) Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin. (As an exercise, use two methods to solve this question. You can view z as a function of x and y, or you can use the Lagrange Multipliers.)

(6) A model for the yield Y of an agricultural crop as a function of the nitrogen level N and phosphorus level P in the soil (measured in appropriate units) is

$$Y(N, P) = kNPe^{-N-4P}$$

where k is a positive constant. What levels of nitrogen and phosphorus result in the best yield?

(7) *Find the points on both the plane $x+y+2z = 2$ and the paraboloid $z = x^2+y^2$ that are nearest to and farthest from the origin.

(8) **Find the maximum and minimum values of

$$f(x, y, z) = ye^{x-z}$$

subject to the restrictions $\begin{cases} 9x^2 + 4y^2 + 36z^2 = 36 \\ xy + yz = 1 \end{cases}$.

(9) ***

(a) Find the maximum value of

$$f(x_1, ..., x_n) = \sqrt[n]{x_1 x_2 ... x_n}$$

given that $x_1, x_2, ..., x_n$ are positive numbers and $x_1 + x_2 + ... + x_n = c$, where c is a constant.

(b) Deduce from part (a) that if $x_1, x_2, ..., x_n$ are positive numbers, then

$$\sqrt[n]{x_1 x_2 ... x_n} \leqslant \frac{x_1 + x_2 + ... + x_n}{n}.$$

Under what circumstances the equality holds?