MATH 6A WORKSHEET 7

DANNING LU

1. More on Optimization

Question from last worksheet:

(1) Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2x$ on the set D, which is the closed triangular region with vertices $(2, 0)$, $(0, 2)$ and $(0, -2)$.

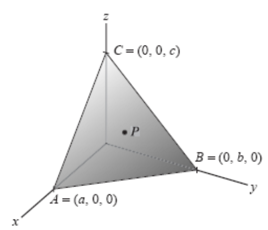
(2) Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin. (As an exercise, use two methods to solve this question. You can view z as a function of x and y, or you can use the Lagrange Multipliers.)

(3) A model for the yield Y of an agricultural crop as a function of the nitrogen level N and phosphorus level P in the soil (measured in appropriate units) is

$$Y(N,P) = kNPe^{-N-4P}$$

where k is a positive constant. What levels of nitrogen and phosphorus result in the best yield? New question:

(4) A plane with equation $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ $(a,b,c > 0)$ together with the positive coordinate planes forms a tetrahedron of volume $V = \frac{1}{6}abc$. Find the plane that minimizes V if the plane is constrained to pass through a point $P(7,4,6)$.



2. Path integral for real-valued functions

Compute $\int_{\mathbf{c}} f\, ds$.

(1) $f(x,y,z) = 2xy - z$, $\mathbf{c}(t) = < 2\sin t, 2\cos t, 7t >$.
(2) $f(x,y) = x^3y^{30}$, $\mathbf{c}$ is the unit circle in $\mathbb{R}^2$.
(3) $f(x,y,z) = x + 2y - z^2$, $\mathbf{c}$ consists of the path $t\mathbf{i} + t^2\mathbf{j}$ from (0,0,0) to (1,1,0), followed by the straight line to $(1,-1,1)$.
(4) $*f(x,y) = x^3 + y^3$, $\mathbf{c}$ is the part of the curve $x^{2/3} + y^{2/3} = 1$ in the first quadrant.