MATH 6A WORKSHEET 8

DANNING LU

Note: the graphs for this answer sheet are available via this link:
https://www.desmos.com/calculator/3nq6ndwhnx

(1) Evaluate the integrals.
   (a) $\int_{-2}^{1}\int_{-1}^{2} 32x^3y^3dxdy$.
   Answer: -450.
   (b) $\int_1^2\int_2^4 \frac{x}{y}dydx$.
   Answer: $\frac{3}{2}\ln 2$.
   (Note: $\ln 4 - \ln 2 = \ln 2$.
   (c) $\iint_D x\cos(2x+y)dA$ where D is the region $0 \leqslant x \leqslant \pi/3$, $0 \leqslant y \leqslant \pi/4$.
   Answer: $\frac{1}{2}(-\frac{\pi}{6} - \frac{\pi}{3}\cos(\frac{11\pi}{12}) + \frac{\sqrt{3}}{2}(-\frac{1}{2} + \cos(\frac{7\pi}{12})))$

(2) Evaluate the integrals.
   (a) $\int_0^2\int_0^x (x+2y)dydx$.
   Answer: 16/3.
   Area of integration: See desmos under 2/a.
   (b) $\int_0^1\int_{1-x}^{1+x}(24x^2+4y)dydx$. Answer: 16.
   Area of integration: See desmos under 2/b.
   (c) $\iint_D xydA$ where D is the triangle with vertices (0,0),(6,0),(0,1).
   Answer: 3/2.
   Possible form for double integral:

$$\int_0^6\int_0^{1-\frac{1}{6}x} xydydx$$

   or

$$\int_0^1\int_0^{6-6y} xydxdy$$

(3) Compute the solid under the graph of $f(x,y) = 3 + 2x^2 + 7y$ over the rectangle $R = \{(x,y)|1 \le x \le 3, 0 \le y \le 1\}$.
   Answer: $\int_0^1\int_1^3(3+2x^2+7y)dxdy = 91/3$.

(4) Reverse order of integration.
   (a) $\int_0^1\int_x^{2x} e^{y-x}dydx$.
   Answer: Graph: See desmos under 4/a.

$$\int_0^1\int_{y/2}^{y} e^{y-x}dydx + \int_1^2\int_{y/2}^{1} e^{y-x}dydx$$

(b) $\int_0^{2\sqrt{3}} \int_{y^2/6}^{\sqrt{16-y^2}} 1dxdy$.
Answer: Graph: See desmos under 4/b.

$$\int_0^2 \int_0^{\sqrt{6x}} 1dydx + \int_2^4 \int_0^{\sqrt{16-x^2}} 1dydx$$

(c) $\int_0^7 \int_{x^2-6x}^{x} f(x,y)dydx$.
Answer: Graph: see desmos under 4/c.

$$\int_{-9}^0 \int_{3-\sqrt{9+y}}^{3+\sqrt{9+y}} f(x,y)dxdy + \int_0^7 \int_x^{3+\sqrt{9+y}} f(x,y)dxdy$$

(d) $\int_1^2 \int_x^{x^3} f(x,y)dydx + \int_2^8 \int_x^8 f(x,y)dydx$.
Answer: Graph: see desmos under 4/d.

$$\int_1^8 \int_{\sqrt[3]{y}}^{y} f(x,y)dxdy$$

(5) Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{7y}^7 e^{x^2} dxdy.$$

Answer: Graph: see desmos under 5.

$$\int_0^7 \int_0^{x/7} e^{x^2} dydx = \int_0^7 \frac{xe^{x^2}}{7} dx = \frac{e^{49}-1}{14}$$

QUIZZES

NAME:________________________
PERM:_________________________

Show your work. Points will NOT be awarded for answers with no explanation or necessary steps. NO CALCULATORS. NO NOTES.

(1) Evaluate the line integral $\int_c f\, ds$, where $f(x,y) = x^3y^{18}$ and c is the upper half of the unit circle centred at the origin (i.e., the part of the unit circle that is above the y-axis). (4 points)

Answer:
Method 1: Use $\mathbf{c}(t) = \langle \cos t, \sin t\rangle$ with $0 \leqslant x \leqslant \pi$. Then $\mathbf{c}'(t) = \langle -\sin t, \cos t\rangle$ and so $||\mathbf{c}'(t)|| = 1$. The integral will become

$$\int_0^\pi \cos^3(t)\sin^{18}(t)\cdot 1dt.$$

By using $u = \sin t$ and using the identity $\cos^2 t = 1 - \sin^2 t$, we get that the integral becomes

$$\int_0^0 (1-u^2)u^{18}du = 0.$$

(Be careful we have to change the bounds.)
Method 2: We can discover that the integral of the left part of the semicircle will be negative of that of the right part, (since the value of the function at $(-x,y)$ is exactly the negative of the function at (x,y), and that the semicircle is symmetric about the y-axis), we know that the integral must be 0.

(2) Evaluate $\int_c \mathbf{F}\cdot ds$, where c is the line connecting $(0,-\pi)$ and (0,0), and $\mathbf{F} = \langle e^x \sin y, e^x\cos y\rangle$. (4 points)
Answer: Possible parametric function: $\mathbf{c}(t) = \langle 0, t\rangle$, $-\pi \leqslant t \leqslant 0$. Thus $\mathbf{c}'(t) = \langle 0, 1\rangle$

$$\int_c \mathbf{F}\cdot ds = \int_{-\pi}^0 \langle e^0\sin t, e^0\cos t\rangle \cdot \langle 0,1\rangle dt = \int_{-\pi}^0 e^0 \cos t dt = 0.$$

(3) Evaluate $\int_c \mathbf{F}\cdot ds$, where $c(t) = \langle t\sin t, t\cos t\rangle$, $0 \leqslant t \leqslant \pi$, and $\mathbf{F} = \langle e^x\sin y, e^x\cos y\rangle$. (2 points)
Answer: Discover that $\mathbf{F}$ is conservative by verifying

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} = e^x\cos y,$$

we know that the integral does not depend on the path. Since in this problem $\mathbf{c}(0) = \langle 0,0\rangle$, $\mathbf{c}(\pi) = \langle 0,-\pi\rangle$, we can just choose the inverse path in the previous question, and hence by the property of line integral for vector fields, we know that the answer will just be the negative of the answer of the previous question, which is still 0.