MATH 6A WORKSHEET 10

DANNING LU

1. Surface Integrals

Let S be the unit sphere $x^2 + y^2 + z^2 = 1$.

(1) Compute the surface integral $\iint_S (x + y) dS$.

(2) Compute the surface integral $\iint_S \langle x, y, z \rangle d\mathbf{S}$.

(3) Compute the surface integral $\iint_S \langle x^2, y^2, z^2 \rangle d\mathbf{S}$.

(4) A coffee filter is shaped like a cone described in cylindrical coordinates by $z = r$, $0 \leqslant r \leqslant 4$ (distance is measured in cm). Coffee flows straight downward, and flows faster through the center of the filter. In fact, the ow of coffee is described by the vector field $\mathbf{F} = \langle 0, 0, r - 4 \rangle$, measured in cm/min. Find the rate (in cm3/min) at which coffee flows through the entire filter by calculating the flux.

2. Green's theorem, Divergence theorem and Stroke's theorem

(1) Write down the three theorems.

(2) Use Green's Theorem to find the circulation of $\mathbf{F}(x, y) = \langle x^3, y^3 \rangle$ along the counterclockwise loop C consisting of the two line segments on the x- and y-axis with $0 \leqslant x \leqslant 2$ and $0 \leqslant y \leqslant 2$, respectively, and the quarter circle centred at origin with radius two in the first quadrant?

(3) Let U be the surface $z = xy$ where $-1 \leqslant x \leqslant 1$, $-1 \leqslant y \leqslant 1$. Let C be the quadrilateral from $(1, 1, 1)$ to $(-1, 1, -1)$ to $(-1, -1, 1)$ to $(1, -1, -1)$, and back to $(1, 1, 1)$, so that C is the boundary of U. Use Stokes' Theorem to fi
nd the work done by $\mathbf{F}(x, y, z) = yz^2\vec{i} + xz^2\vec{j}$ along the loop C.

(4) Let U be the surface shown where $z = \sin(x)\sin(y)$, where $0 \leqslant x \leqslant \pi$, $0 \leqslant y \leqslant \pi$, and let $\mathbf{F}(x, y, z) = z\vec{k}$. Notice that the boundary of U is a square in the xy-plane.
   (a) Let T be the square region in the xy plane where $0 \leqslant x \leqslant \pi$, $0 \leqslant y \leqslant \pi$. Find the flux of $\mathbf{F}$ down through T.
   (b) Let E be the solid region between U and T, so that U and T together form the boundary (or surface) of E. Find $\iiint_E div(\mathbf{F})dV$.
   (c) Use the results above to find $\iint_U \mathbf{F} \cdot d\vec{S}$. Explain your work.