MATH 6A WORKSHEET 9 ANSWER

DANNING LU

1. More on Double Integrals

(1) Find Volume of solid.
  (a) Tetrahedron in first octant bounded by coordinate planes and $z = 7 - 3x - 2y$.
  Answer:
  $$\int 0^{7/2} \int_0^{\frac{7-2y}{3}} (7 - 3x - 2y)dxdy = 343/36.$$

  (b) Solid inside both the sphere $x^2 + y^2 + z^2 = 3$ and paraboloid $2z = x^2 + y^2$.
  $$\int_0^{2\pi} \int_0^{\sqrt{2}} (\sqrt{3 - r^2} - \frac{r^2}{2})rdrd\theta = 2\pi(\sqrt{3} - 5/6).$$

(2) Compute $\int_0^{\pi/3} \int_1^3 re^{-r^2} drd\theta$. If this represents $\iint_D f(x, y)dA$, find the function $f(x, y)$ and the region D.
  Explained in class.

(3) (a) Evaluate $\iint_{\mathbb{R}^2} e^{-x^2 - y^2} dA$.
  (b) Use the above result to find out the value of the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$.
  Explained in class

(4) Find area inside both $r = 1$ and $r = 2\sin\theta$.

$$\int_0^{\pi/6} \int_0^{2\sin\theta} rdrd\theta + \int_{\pi/6}^{5\pi/6} \int_0^1 rdrd\theta + \int_{5\pi/6}^{\pi} \int_0^{2\sin\theta} rdrd\theta = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

2. Triple Integrals in Cartesian Coordinate

Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

(1) $f = xy + z^2$. $E = \{(x, y, z) | 0 \le x \le 2, 0 \le y \le 1, 0 \le z \le 3\}$.
  Answer:
  $$\int_0^2 \int_0^1 \int_0^3 (xy + z^2)dzdydx = 21$$

(2) $f = y$. $E = \{(x, y, z) | 0 \le x \le 3, 0 \le y \le x, x - y \le z \le x + y\}$.
  Answer:
  $$\int_0^3 \int_0^x \int_{x-y}^{x+y} ydzdydx = 27/2$$

(3) $f = \sin y$. E is the plane below $z = x$ and above the triangle region with vertices $(0, 0, 0), (0, \pi, 0), (\pi, 0, 0)$.

Answer:
$$\int_0^\pi \int_0^{\pi-x} \int_0^x \sin y \, dz dy dx = \pi^2/2 - 2$$

(4) $f = x - y$. E is enclosed by the surfaces $z = x^2 - 1$, $z = 1 - x^2$, $y = 0$ and $y = 2$.
$$\int_0^2 \int_{-1}^1 \int_{x^2-1}^{1-x^2} (x - y) dz dx dy = -16/3$$

(5) $f = xz$. E is the tetrahedron with vertices $(0, 0, 0), (0, 1, 0), (0, 0, 2), (3, 0, 0)$.
6-6y-3z-2x=0
$$\int_0^2 \int_0^{1-z/2} \int_0^{3-3y-3z/2} xz dx dy dz = 3/10$$

Part 3 and 4 are omitted here since it will probably not be on the final. We are going to discuss the problems in part 5 together in week 10.

QUIZZES

NAME:_____ PERM:_____ SECTION TIME:_____

Show your work. Partial points might be awarded. NO CALCULATORS. NO NOTES.

(1) Evaluate $\iint_D xy\,dA$, where D is the triangle region with vertices $(0,1)$, $(1,2)$, $(4,1)$.

Answer:

$$\int_1^2 \int_{y-1}^{7-3y} xy\,dx\,dy = 13/3.$$

Note:

$$\int_0^1 \int_1^{x+1} xy\,dy\,dx + \int_1^4 \int_1^{\frac{7-x}{3}} xy\,dy\,dx$$

will also work, but the computation will be much more difficult.

(2) Evaluate $\iint_R (x + \arctan \frac{y}{x})dA$, where $R = \{(x,y)|1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

Answer:

$$\int_0^{\pi/4} \int_1^2 (r\cos\theta + \theta)r\,dr\,d\theta = \frac{7}{3\sqrt{2}} + \frac{3\pi^2}{64}$$