MATH 6A WORKSHEET 8

DANNING LU

1. More on Double Integrals

(1) Find Volume of solid.
   (a) Tetrahedron in first octant bounded by coordinate planes and $z = 7 - 3x - 2y$.

   (b) Solid inside both the sphere $x^2 + y^2 + z^2 = 3$ and paraboloid $2z = x^2 + y^2$.

(2) Compute $\int_0^{\pi/3} \int_1^3 re^{-r^2} dr d\theta$. If this represents $\iint_D f(x, y) dA$, find the function $f(x, y)$ and the region D.

(3) (a) Evaluate $\iint_{\mathbb{R}^2} e^{-x^2-y^2} dA$.

   (b) Use the above result to find out the value of the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$.

(4) Find area inside both $r = 1$ and $r = 2\sin\theta$.

2. Triple Integrals in Cartesian Coordinate

Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

(1) $f = xy + z^2$. $E = \{(x, y, z)|0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq 3\}$.

(2) $f = y$. $E = \{(x, y, z)|0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y\}$.

(3) $f = \sin y$. E is the plane below $z = x$ and above the triangle region with vertices $(0, 0, 0), (0, \pi, 0), (\pi, 0, 0)$.

(4) $f = x - y$. E is enclosed by the surfaces $z = x^2 - 1$, $z = 1 - x^2$, $y = 0$ and $y = 2$.

(5) $f = xz$. E is the tetrahedron with vertices $(0,0,0), (0,1,0), (0,0,2), (3,0,0)$.

3. **Triple Integral in Cylindrical Coordinate

Sketch the solid and function that is being integrated by the formula given below.

(1) $\int_{-\pi/2}^{\pi/2} \int_0^2 \int_0^{r^2} z dz dr d\theta$.

(2) $\int_0^2 \int_0^{2\pi} \int_0^r rz \sin\theta dz d\theta dr$.

Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

(1) $f = \sqrt{x^2 + y^2}$. E is the solid that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

(2) $f = z$. E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

(3) $f = x + y + z$. E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

4. **Triple Integral in Spherical Coordinate

Sketch the solid and function that is being integrated by the formula given below.

(1) $\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho d\rho d\theta d\phi$.

(2) $\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec\phi} \rho\cos\theta\sin\phi d\rho d\theta d\phi$.

Evaluate the integral $\iiint_E f dV$ with f and E given below. You may need to draw the region for your integral.

(1) $f = y^2z^2$. E is the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 1$.

(2) $f = xe^{x^2+y^2+z^2}$. E is the solid in the first octant and between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 8$.

Quizzes

NAME:_____ PERM:_____ SECTION TIME:_____
Show your work. Partial points might be awarded. NO CALCULATORS. NO NOTES.

(1) Evaluate $\iint_D xydA$, where D is the triangle region with vertices $(0,1)$, $(1,2)$, $(4,1)$.

(2) Evaluate $\iint_R (x + \arctan\frac{y}{x})dA$, where $R = \{(x,y)|1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.