Math 3C

Lecture I

Intro to differential equations

Familiar example:

$$y(t) = e^{2t}$$
$$y'(t) = 2e^{2t} = 2y(t)$$

what does it mean to be given an "equation"

$$y' = 2y$$

and ask what its solution is? What kind of an equation is this, and how do we solve it?

Algebraic Equations

These have numbers and variables, which are arbitrary numbers

$$5x + 8 = -3x$$

we solve them using algebraic rules to find $x = -1$

And we know we are right since plugging -1 in for x gives a tautology $0 = 0$.

Note the algebraic equations

$x^2 - 1 = 0$ has two solutions, $x = \pm 1$

$x - x = 3$ has no solutions

Differential Equations

These too have numbers and variables, except now the variables represent functions and their derivatives.

eg. $y' = 2y$

Unfortunately, there is no simple analog to the "algebraic rules" we used earlier to reduce the equation and find an answer, but we can still plug in a guess and see if we arrive at a tautology.

Indeed if we guess $y = e^{2t}$ and plug in we get

$$2e^{2t} = 2e^{2t} \quad \checkmark$$

What if we guessed $y = e^{3t}$? Then plugging in gives

$$3e^{3t} = 2e^{3t} \Rightarrow 3 = 2 \quad X$$

What about $y = e^{2t} + 7$? Then $y' = 2y \Rightarrow 2e^{2t} = 2(e^{2t} + 7) = 2e^{2t} + 14 \quad X$

What about this guess: $y = 7e^{2t}$, plugging in gives

$$14e^{2t} = 14e^{2t} \checkmark$$

In fact $y = Ae^{2t}$ where A is any # is a solution to the DE

So DE's usually have an infinite number of solutions, which tend to come in families, unlike algebraic equations which have a finite # of solutions.

Here's another DE

$$y'' = -y$$

guess ① $y = \cos(t)$

② $y = \sin(t)$

③ $y = 3\cos(t) + 2\sin(t)$