

MATH 116

Midterm

50 points

Instructions: Read each question carefully. Each problem is worth **10 points**. Write your solutions on the colored paper. Do not turn in scratch work. Good luck.

1. There are n married couples at a party. Each person shakes hands with every person other than his or her spouse. Find the total number of handshakes.
2. Let S be the set $S = \{1, 2, \dots, 10\}$. Find the number of subsets of S that contain the number 1, 3, or 7.
3. We are to seat five boys, five girls, and one parent in at a circular table with eleven seats. In how many ways can this be done if no boy is to sit next to a boy and no girl is to sit next to a girl?
4. How many sets of three integers between 1 and 20 are possible if no two consecutive integers are to be in a set?
5. Let n be a positive integer, and let S be an n -element set. If A is a subset of S , denote by $o(A)$ the number of elements in A (in particular, $o(S) = n$). Say that A is *odd* if $o(A)$ is odd, and A is *even* if $o(A)$ is even (the integer 0 is considered even). Prove that the number of odd subsets of S equals the number of its even subsets. Hint: Consider the cases n odd and n even separately.